# Problems of the 2nd and 9th International Physics Olympiads (Budapest, Hungary, 1968 and 1976)

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**Abstract**

After a short introduction the problems of the 2nd and the 9th International Physics Olympiad, organized in Budapest, Hungary, 1968 and 1976, and their solutions are presented.

## Introduction

Following the initiative of Dr. Waldemar Gorzkowski [1] I present the problems and solutions of the 2nd and the 9th International Physics Olympiad, organized by Hungary. I have used Prof. Rezső Kunfalvi's problem collection [2], its Hungarian version [3] and in the case of the 9th Olympiad the original Hungarian problem sheet given to the students (my own copy). Besides the digitalization of the text, the equations and the figures it has been made only small corrections where it was needed (type mistakes, small grammatical changes). I omitted old units, where both old and SI units were given, and converted them into SI units, where it was necessary.

If we compare the problem sheets of the early Olympiads with the last ones, we can realize at once the difference in length. It is not so easy to judge the difficulty of the problems, but the solutions are surely much shorter.

The problems of the 2nd Olympiad followed the more than hundred years tradition of physics competitions in Hungary. The tasks of the most important Hungarian theoretical physics competition (Eötvös Competition), for example, are always very short. Sometimes the solution is only a few lines, too, but to find the idea for this solution is rather difficult.

Of the 9th Olympiad I have personal memories; I was the youngest member of the Hungarian team. The problems of this Olympiad were collected and partly invented by Miklós Vermes, a legendary and famous Hungarian secondary school physics teacher. In the first problem only the detailed investigation of the stability was unusual, in the second problem one could forget to subtract the work of the atmospheric pressure, but the fully "open" third problem was really unexpected for us.

The experimental problem was difficult in the same way: in contrast to the Olympiads of today we got no instructions how to measure. (In the last years the only similarly open experimental problem was the investigation of "The magnetic puck" in Leicester, 2000, a really nice problem by Cyril Isenberg.) The challenge was not to perform many-many measurements in a short time, but to find out what to measure and how to do it.

Of course, the evaluating of such open problems is very difficult, especially for several hundred students. But in the 9th Olympiad, for example, only ten countries participated and the same person could read, compare, grade and mark all of the solutions.

# 2nd IPhO (Budapest, 1968)

## Theoretical problems

### Problem 1

On an inclined plane of 30° a block, mass $m_2$ = 4 kg, is joined by a light cord to a solid cylinder, mass $m_1$ = 8 kg, radius $r$ = 5 cm (*Fig. 1*). Find the acceleration if the bodies are released. The coefficient of friction between the block and the inclined plane $\mu = 0.2$. Friction at the bearing and rolling friction are negligible.

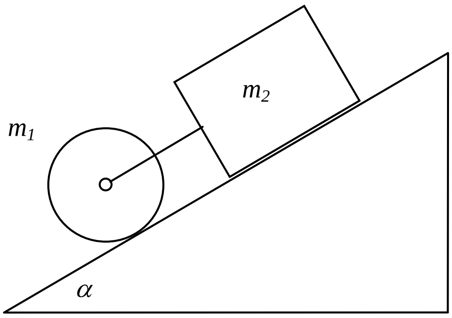


*Figure 1*

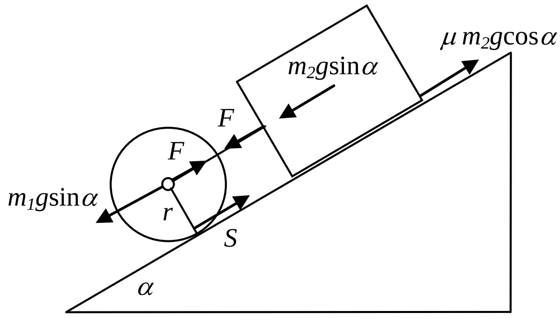


*Figure 2*

### Solution

If the cord is stressed the cylinder and the block are moving with the same acceleration $a$. Let $F$ be the tension in the cord, $S$ the frictional force between the cylinder and the inclined plane (*Fig. 2*). The angular acceleration of the cylinder is $a/r$. The net force causing the acceleration of the block:

$$m_2 a = m_2 g\sin\alpha - \mu m_2 g\cos\alpha + F\,,$$

and the net force causing the acceleration of the cylinder:

$$m_1 a = m_1 g\sin\alpha - S - F\,.$$

The equation of motion for the rotation of the cylinder:

$$S\,r = \frac{a}{r}\cdot I\,.$$

($I$ is the moment of inertia of the cylinder, $S\cdot r$ is the torque of the frictional force.)
Solving the system of equations we get:

$$a = g\cdot\frac{(m_1+m_2)\sin\alpha - \mu m_2\cos\alpha}{m_1+m_2+\dfrac{I}{r^2}}\,, \qquad (1)$$

$$S = \frac{I}{r^2}\cdot g\cdot\frac{(m_1+m_2)\sin\alpha - \mu m_2\cos\alpha}{m_1+m_2+\dfrac{I}{r^2}}\,, \qquad (2)$$

$$F = m_2 g \cdot \frac{\mu\left(m_1 + \frac{I}{r^2}\right)\cos\alpha - \frac{I\sin\alpha}{r^2}}{m_1 + m_2 + \frac{I}{r^2}} . \tag{3}$$

The moment of inertia of a solid cylinder is $I = \frac{m_1 r^2}{2}$. Using the given numerical values:

$$a = g \cdot \frac{(m_1 + m_2)\sin\alpha - \mu m_2 \cos\alpha}{1.5 m_1 + m_2} = 0.3317g = \mathbf{3.25\ m/s^2} ,$$
$$S = \frac{m_1 g}{2} \cdot \frac{(m_1 + m_2)\sin\alpha - \mu m_2 \cos\alpha}{1.5 m_1 + m_2} = \mathbf{13.01\ N} ,$$
$$F = m_2 g \cdot \frac{(1.5\mu\cos\alpha - 0.5\sin\alpha)m_1}{1.5 m_1 + m_2} = \mathbf{0.192\ N} .$$

**Discussion** (See *Fig. 3*.)

The condition for the system to start moving is $a > 0$. Inserting $a = 0$ into (1) we obtain the limit for angle $\alpha_1$:

$$\tan\alpha_1 = \mu \cdot \frac{m_2}{m_1 + m_2} = \frac{\mu}{3} = 0.0667 , \quad \alpha_1 = 3.81° .$$

For the cylinder separately $\alpha_1 = 0$, and for the block separately $\alpha_1 = \tan^{-1}\mu = 11.31°$.

If the cord is not stretched the bodies move separately. We obtain the limit by inserting $F = 0$ into (3):

$$\tan\alpha_2 = \mu \cdot \left(1 + \frac{m_1 r^2}{I}\right) = 3\mu = 0.6 , \quad \alpha_2 = 30.96° .$$

The condition for the cylinder to slip is that the value of $S$ (calculated from (2) taking the same coefficient of friction) exceeds the value of $\mu m_1 g\cos\alpha$. This gives the same value for $\alpha_3$ as we had for $\alpha_2$. The acceleration of the centers of the cylinder and the block is the same: $g(\sin\alpha - \mu\cos\alpha)$, the frictional force at the bottom of the cylinder is $\mu m_1 g\cos\alpha$, the peripheral acceleration of the cylinder is $\mu \cdot \frac{m_1 r^2}{I} \cdot g\cos\alpha$.

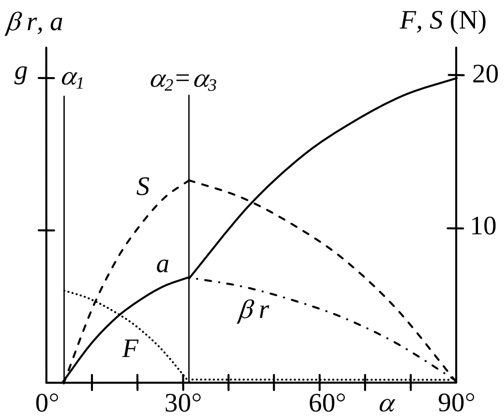

*Figure 3*

**Problem 2**

There are 300 cm$^3$ toluene of 0°C temperature in a glass and 110 cm$^3$ toluene of 100°C temperature in another glass. (The sum of the volumes is 410 cm$^3$.) Find the final volume after the two liquids are mixed. The coefficient of volume expansion of toluene $\beta = 0.001(°\mathrm{C})^{-1}$. Neglect the loss of heat.

**Solution**

If the volume at temperature $t_1$ is $V_1$, then the volume at temperature 0°C is $V_{10} = V_1/(1+\beta t_1)$. In the same way if the volume at $t_2$ temperature is $V_2$, at 0°C we have $V_{20} = V_2/(1+\beta t_2)$. Furthermore if the density of the liquid at 0°C is $d$, then the masses are $m_1 = V_{10}d$ and $m_2 = V_{20}d$, respectively. After mixing the liquids the temperature is

$$t = \frac{m_1 t_1 + m_2 t_2}{m_1 + m_2}.$$

The volumes at this temperature are $V_{10}(1+\beta t)$ and $V_{20}(1+\beta t)$.
The sum of the volumes after mixing:

$$\begin{aligned} &V_{10}(1+\beta t) + V_{20}(1+\beta t) = V_{10} + V_{20} + \beta(V_{10} + V_{20})t = \\ &= V_{10} + V_{20} + \beta \cdot \frac{m_1 + m_2}{d} \cdot \frac{m_1 t_1 + m_2 t_2}{m_1 + m_2} = \\ &= V_{10} + V_{20} + \beta\left(\frac{m_1 t_1}{d} + \frac{m_2 t_2}{d}\right) = V_{10} + \beta V_{10} t_1 + V_{20} + \beta V_{20} t_2 = \\ &= V_{10}(1+\beta t_1) + V_{20}(1+\beta t_2) = V_1 + V_2 \end{aligned}$$

The sum of the volumes is constant. In our case it is 410 cm$^3$. The result is valid for any number of quantities of toluene, as the mixing can be done successively adding always one more glass of liquid to the mixture.

**Problem 3**

Parallel light rays are falling on the plane surface of a semi-cylinder made of glass, at an angle of 45°, in such a plane which is perpendicular to the axis of the semi-cylinder (*Fig. 4*). (Index of refraction is $\sqrt{2}$.) Where are the rays emerging out of the cylindrical surface?

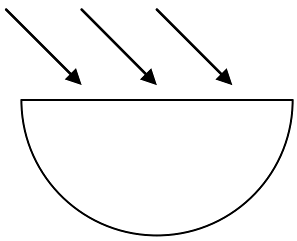

*Figure 4*

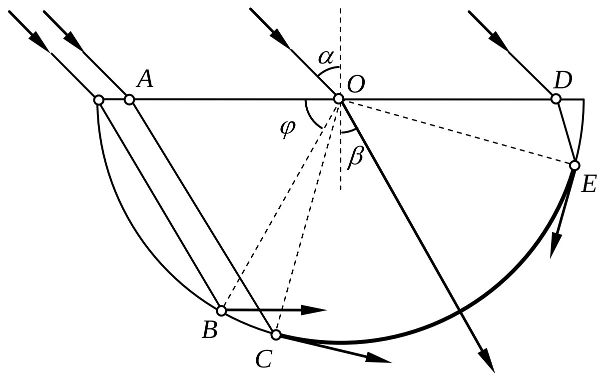


*Figure 5*

**Solution**

Let us use angle $\varphi$ to describe the position of the rays in the glass (*Fig. 5*). According to the law of refraction $\sin 45°/\sin\beta = \sqrt{2}$, $\sin\beta = 0.5$, $\beta = 30°$. The refracted angle is 30° for all of the incoming rays. We have to investigate what happens if $\varphi$ changes from 0° to 180°.

It is easy to see that $\varphi$ can not be less than 60° ( $AOB\angle = 60°$). The critical angle is given by $\sin\beta_{crit} = 1/n = \sqrt{2}/2$; hence $\beta_{crit} = 45°$. In the case of total internal reflection $ACO\angle = 45°$, hence $\varphi = 180° - 60° - 45° = 75°$. If $\varphi$ is more than 75° the rays can emerge the cylinder. Increasing the angle we reach the critical angle again if $OED\angle = 45°$. Thus the rays are leaving the glass cylinder if:

$$75° < \varphi < 165°,$$

CE, arc of the emerging rays, subtends a central angle of 90°.

**Experimental problem**

Three closed boxes (black boxes) with two plug sockets on each are present for investigation. The participants have to find out, without opening the boxes, what kind of elements are in them and measure their characteristic properties. AC and DC meters (their internal resistance and accuracy are given) and AC (5O Hz) and DC sources are put at the participants' disposal.

**Solution**

No voltage is observed at any of the plug sockets therefore none of the boxes contains a source.

Measuring the resistances using first AC then DC, one of the boxes gives the same result. Conclusion: the box contains a simple resistor. Its resistance is determined by measurement.

One of the boxes has a very great resistance for DC but conducts AC well. It contains a capacitor, the value can be computed as $C = \dfrac{1}{\omega X_C}$.

The third box conducts both AC and DC, its resistance for AC is greater. It contains a resistor and an inductor connected in series. The values of the resistance and the inductance can be computed from the measurements.

# $9^{th}$ IPhO (Budapest, 1976)

## Theoretical problems

### Problem 1

A hollow sphere of radius $R$ = 0.5 m rotates about a vertical axis through its centre with an angular velocity of $\omega$ = 5 $s^{-1}$. Inside the sphere a small block is moving together with the sphere at the height of $R/2$ (*Fig. 6*). ($g$ = 10 m/$s^2$.)

a) What should be at least the coefficient of friction to fulfill this condition?

b) Find the minimal coefficient of friction also for the case of $\omega$ = 8 $s^{-1}$.

c) Investigate the problem of stability in both cases,

α) for a small change of the position of the block,

β) for a small change of the angular velocity of the sphere.

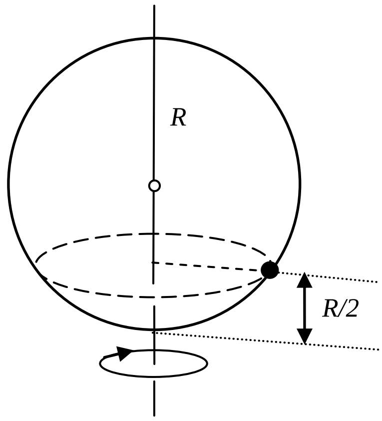


*Figure 6*

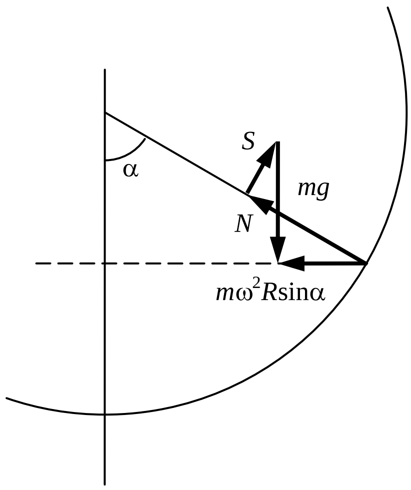


*Figure 7*

### Solution

a) The block moves along a horizontal circle of radius $R\sin\alpha$. The net force acting on the block is pointed to the centre of this circle (*Fig. 7*). The vector sum of the normal force exerted by the wall $N$, the frictional force $S$ and the weight $mg$ is equal to the resultant: $m\omega^2 R\sin\alpha$.

The connections between the horizontal and vertical components:

$$m\omega^2 R\sin\alpha = N\sin\alpha - S\cos\alpha\ ,$$

$$mg = N\cos\alpha + S\sin\alpha\ .$$

The solution of the system of equations:

$$S = mg\sin\alpha\left(1 - \frac{\omega^2 R\cos\alpha}{g}\right),$$

$$N = mg\left(\cos\alpha + \frac{\omega^2 R\sin^2\alpha}{g}\right).$$

The block does not slip down if

$$\mu_a \geq \frac{S}{N} = \sin\alpha \cdot \frac{1 - \dfrac{\omega^2 R\cos\alpha}{g}}{\cos\alpha + \dfrac{\omega^2 R\sin^2\alpha}{g}} = \frac{3\sqrt{3}}{23} = \mathbf{0.2259}\,.$$

In this case there must be at least this friction to prevent slipping, i.e. sliding down.

b) If on the other hand $\dfrac{\omega^2 R\cos\alpha}{g} > 1$ some friction is necessary to prevent the block to slip upwards. $m\omega^2 R\sin\alpha$ must be equal to the resultant of forces *S*, *N* and *mg*. Condition for the minimal coefficient of friction is (*Fig. 8*):

$$\mu_b \geq \frac{S}{N} = \sin\alpha \cdot \frac{\dfrac{\omega^2 R\cos\alpha}{g} - 1}{\cos\alpha + \dfrac{\omega^2 R\sin^2\alpha}{g}} =$$

$$= \frac{3\sqrt{3}}{29} = \mathbf{0.1792}\,.$$

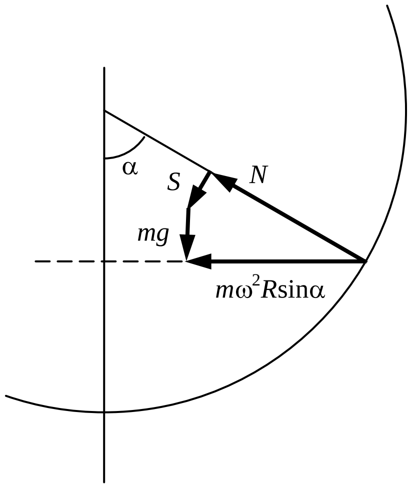


*Figure 8*

c) We have to investigate $\mu_a$ and $\mu_b$ as functions of $\alpha$ and $\omega$ in the cases a) and b) (see *Fig. 9/a* and *9/b*):

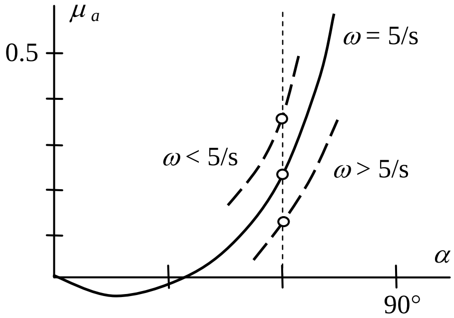


*Figure 9/a*

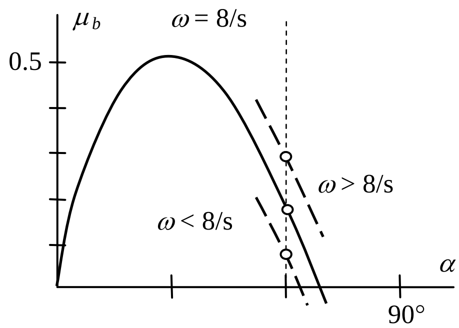


*Figure 9/b*

In case a): if the block slips upwards, it comes back; if it slips down it does not return. If $\omega$ increases, the block remains in equilibrium, if $\omega$ decreases it slips downwards.

In case b): if the block slips upwards it stays there; if the block slips downwards it returns. If $\omega$ increases the block climbs upwards-, if $\omega$ decreases the block remains in equilibrium.

## Problem 2

The walls of a cylinder of base 1 dm$^2$, the piston and the inner dividing wall are perfect heat insulators (*Fig. 10*). The valve in the dividing wall opens if the pressure on the right side is greater than on the left side. Initially there is 12 g helium in the left side and 2 g helium in the right side. The lengths of both sides are 11.2 dm each and the temperature is

$0°C$. Outside we have a pressure of 100 kPa. The specific heat at constant volume is $c_v = 3.15$ J/gK, at constant pressure it is $c_p = 5.25$ J/gK. The piston is pushed slowly towards the dividing wall. When the valve opens we stop then continue pushing slowly until the wall is reached. Find the work done on the piston by us.

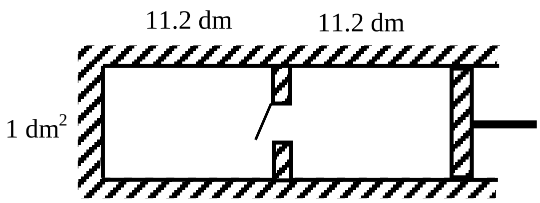


*Figure 10*

**Solution**

The volume of 4 g helium at $0°C$ temperature and a pressure of 100 kPa is 22.4 dm$^3$ (molar volume). It follows that initially the pressure on the left hand side is 600 kPa, on the right hand side 100 kPa. Therefore the valve is closed.

An adiabatic compression happens until the pressure in the right side reaches 600 kPa ($\kappa = 5/3$).

$$100 \cdot 11.2^{5/3} = 600 \cdot V^{5/3},$$

hence the volume on the right side (when the valve opens):

$$V = 3.82 \text{ dm}^3.$$

From the ideal gas equation the temperature is on the right side at this point

$$T_1 = \frac{pV}{nR} = 552\text{K}.$$

During this phase the whole work performed increases the internal energy of the gas:

$$W_1 = (3.15 \text{ J/gK}) \cdot (2 \text{ g}) \cdot (552 \text{ K} - 273 \text{ K}) = 1760 \text{ J}.$$

Next the valve opens, the piston is arrested. The temperature after the mixing has been completed:

$$T_2 = \frac{12 \cdot 273 + 2 \cdot 552}{14} = 313\text{K}.$$

During this phase there is no change in the energy, no work done on the piston.

An adiabatic compression follows from 11.2 + 3.82 = 15.02 dm$^3$ to 11.2 dm$^3$:

$$313 \cdot 15.02^{2/3} = T_3 \cdot 11.2^{2/3},$$

hence

$$T_3 = 381 \text{ K}.$$

The whole work done increases the energy of the gas:

$$W_3 = (3.15 \text{ J/gK}) \cdot (14 \text{ g}) \cdot (381 \text{ K} - 313 \text{ K}) = 3000 \text{ J}.$$

The total work done:

$$W_{total} = W_1 + W_3 = 4760 \text{ J}.$$

The work done by the outside atmospheric pressure should be subtracted:

$$W_{atm} = 100 \text{ kPa} \cdot 11.2 \text{ dm}^3 = 1120 \text{ J}.$$

The work done on the piston by us:

$$W = W_{total} - W_{atm} = \mathbf{3640 \text{ J}}.$$

**Problem 3**

Somewhere in a glass sphere there is an air bubble. Describe methods how to determine the diameter of the bubble without damaging the sphere.

**Solution**

We can not rely on any value about the density of the glass. It is quite uncertain. The index of refraction can be determined using a light beam which does not touch the bubble. Another method consists of immersing the sphere into a liquid of same refraction index: its surface becomes invisible.

A great number of methods can be found.

We can start by determining the axis, the line which joins the centers of the sphere and the bubble. The easiest way is to use the "tumbler-over" method. If the sphere is placed on a horizontal plane the axis takes up a vertical position. The image of the bubble, seen from both directions along the axis, is a circle.

If the sphere is immersed in a liquid of same index of refraction the spherical bubble is practically inside a parallel plate (*Fig. 11*). Its boundaries can be determined either by a micrometer or using parallel light beams.

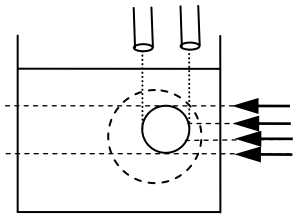

*Figure11*

Along the axis we have a lens system consisting of two thick negative lenses. The diameter of the bubble can be determined by several measurements and complicated calculations.

If the index of refraction of the glass is known we can fit a plano-concave lens of same index of refraction to the sphere at the end of the axis (*Fig. 12*). As ABCD forms a parallel plate the diameter of the bubble can be measured using parallel light beams.

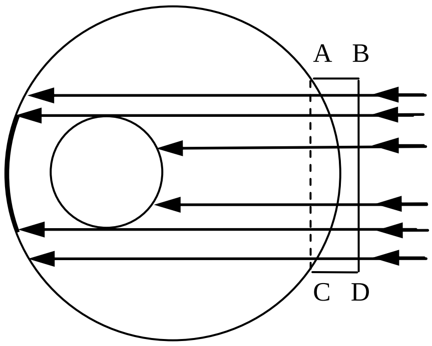


*Figure12*

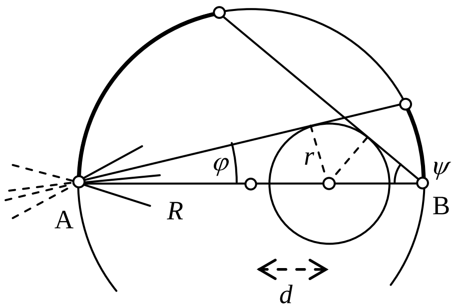


*Figure13*

Focusing a light beam on point A of the surface of the sphere (*Fig. 13*) we get a diverging beam from point A inside the sphere. The rays strike the surface at the other side and illuminate a cap. Measuring the spherical cap we get angle $\varphi$. Angle $\psi$ can be obtained in a similar way at point B. From

$$\sin\varphi = \frac{r}{R+d} \text{ and } \sin\psi = \frac{r}{R-d}$$

we have

$$r = 2R \cdot \frac{\sin\psi \sin\varphi}{\sin\psi + \sin\varphi}, \quad d = R \cdot \frac{\sin\psi - \sin\varphi}{\sin\psi + \sin\varphi}.$$

The diameter of the bubble can be determined also by the help of X-rays. X-rays are not refracted by glass. They will cast shadows indicating the structure of the body, in our case the position and diameter of the bubble.

We can also determine the moment of inertia with respect to the axis and thus the diameter of the bubble.

**Experimental problem**

*The whole text given to the students:*

At the workplace there are beyond other devices a test tube with 12 V electrical heating, a liquid with known specific heat ($c_0$ = 2.1 J/g°C) and an X material with unknown thermal properties. The X material is insoluble in the liquid.

Examine the thermal properties of the X crystal material between room temperature and 70 °C. Determine the thermal data of the X material. Tabulate and plot the measured data.

(You can use only the devices and materials prepared on the table. The damaged devices and the used up materials are not replaceable.)

**Solution**

Heating first the liquid then the liquid and the crystalline substance together two time-temperature graphs can be plotted. From the graphs specific heat, melting point and heat of fusion can be easily obtained.

**Literature**

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