Quantum information processing (BME) / Quantum bits in solids (ELTE)
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Questions

András Pályi[1]

[1]*Department of Physics, Budapest University of Technology and Economics, Hungary*

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The exam will consist of a written and an oral part. Section I below contains a few example exercises, similar to those that will appear in the written part of the exam. In the oral part, I will ask about a specific topic that was covered in the lectures; for example, describe electron-phonon interaction and its consequences with respect to spin-qubit dynamics; or, describe how to control a spin qubit with electric fields, etc. In Section II below, a few further questions are listed; similar to those can also show up during the oral part of the exam.

I. EXAMPLE QUESTIONS FOR THE WRITTEN PART OF THE EXAM

1. List the three Pauli matrices. Determine their eigenvalues and normalized eigenstates.

2. What is the unitary matrix representing the Hadamard gate? What is the result of the Hadamard gate acting on the state $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$? What is the result of the Hadamard gate acting on the state $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$?

3. What is the unitary matrix representing a two-qubit $\sqrt{\text{SWAP}}$ gate in the basis $|00\rangle, |01\rangle, |10\rangle, |11\rangle$? What is the result of the $\sqrt{\text{SWAP}}$ gate acting on the state $\frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$?

4. Determine the polarization vectors associated to the following three states: $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$.

5. Let H be an N-dimensional time independent Hamiltonian, with known energy eigenvalues E_n and eigenstates ψ_n fulfilling $H\psi_n = E_n\psi_n$. Assume that the system is initialized in the state ψ_i at $t = 0$. Express the time evolution of this state, $\psi(t)$, using E_n, ψ_n and ψ_i.

6. Consider the single-qubit state $\frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle$. When we measure the qubit, what is the probability of measuring 1? What is the state of the qubit after the measurement?

7. Consider the two-qubit state $\frac{1}{\sqrt{3}}|00\rangle + \frac{1}{\sqrt{3}}|10\rangle + \frac{1}{\sqrt{3}}|01\rangle$. When we measure the first qubit, what is the probability of measuring 0? And that of measuring 1? What is the state of the system after measuring 0? And after measuring 1?

8. A quantum system is described by the time-dependent Hamiltonian $H(t)$. Write out the time-dependent Schrödinger equation. Perform a time-dependent unitary transformation $U(t)$ on the time-dependent Schrödinger equation. Express the quantity playing the role of the Hamiltonian in the transformed equation, using $H(t)$ and $U(t)$.

9. Consider the Hamiltonian

$$H = H_0 + H_1, \tag{1}$$

where

$$H_0 = \begin{pmatrix} 0 & 0 \\ 0 & \Delta \end{pmatrix} \text{ and } H_1 = \begin{pmatrix} 0 & t \\ t & 0 \end{pmatrix}, \tag{2}$$

with $0 < t \ll \Delta$. Express the lower-energy eigenstate using first-order perturbation theory, and the lower energy eigenvalue using second-order perturbation theory.

10. In GaAs, where the effective mass of the conduction-band electrons is $m^* \approx 0.07 m_e$, we make a quantum dot with an orbital level spacing of 1 meV. Estimate the spatial extension of the electron occupying the ground-state orbital of this quantum dot.

11. It happens sometimes that the writing of the lecturer on the blackboard is unreadable. For example, he might write the spin relaxation rate of an electron in a quasi-one-dimensional quantum dot, due to spin-orbit interaction and spontaneous phonon emission, as

$$\frac{1}{T_1} = \frac{1}{6\pi} \frac{\alpha^2 \omega_L^7}{\rho v_L^? \omega_x^? \hbar^?}, \tag{3}$$

where the question marks denote unreadable integers. Recall the meaning of each quantity of the formula, and use dimensional analysis to determine the values of the unreadable integers.

12. Another unreadable formula on the blackboard encodes the spatial dependence of an inhomogeneous magnetic field in the vicinity of a quantum dot:

$$\boldsymbol{B}(x,y,z) = (\beta ?, 0, B_0), \tag{4}$$

where ? is either x or y. Which one is it? Assume that the lecturer did no mistake.

II. FURTHER EXERCISES

1. Consider an electron in a two-dimensional electron gas, subject to Rashba spin-orbit interaction:

$$H = \frac{p_x^2 + p_y^2}{2m} + \alpha(\sigma_y p_x - \sigma_x p_y). \tag{5}$$

Assume that the electron is occupying a plane-wave state moving along x. Determine the dispersion relation of this electron. How many branches does it have?

2. Consider electrically driven spin resonance due to an ac electric field along the x axis, assisted by an inhomogeneous transverse magnetic field: $\boldsymbol{B} = (0, \beta x, B_0)$. Compare the corresponding spin dynamics to the case of spin-orbit-assisted electrically driven spin resonance.

3. We have discussed a spin relaxation process due to spin orbit interaction and spontaneous phonon emission. Assume that there is no spin-orbit interaction in the quantum dot, but there is hyperfine interaction. Describe the corresponding spin relaxation process: what are the similarities and the differences with respect to the spin-orbit-mediated relaxation? For the discussion, consider the nuclear-spin ensemble as a collection of classical magnetic moments with completely randomized orientations.

4. We have discussed how to drive spin Rabi oscillations using a homogeneous ac electric field in the presence of spin-orbit interaction. Assume now that there is no spin-orbit interaction in the quantum dot, only hyperfine interaction. Describe the corresponding electrically driven spin resonance process: what are the similarities and differences with respect to the spin-orbit mediated dynamics? For the discussion, consider the nuclear-spin ensemble as a collection of classical magnetic moments with completely randomized orientations.

5. In a certain quantum dot, which is hosted in a material where every atomic nucleus carries a nuclear spin, the inhomogeneous dephasing time of the spin of the electron occupying the dot is $T_2^* = 10$ ns. Assume that we can make an identical quantum dot in which 99% of the nuclear spins are eliminated. In this dot, $T_2^* =?$

6. Consider the simplest model of two electrons in a double quantum dot, with two spin states and a single orbital in each dot. What is the dimension of the two-electron Hilbert space? Specify the unitary transformation between the product basis and the singlet-triplet basis. What is the dimension of the three-electron Hilbert space?