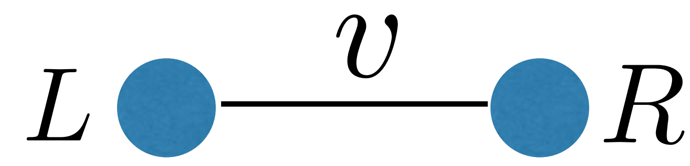
Ehrenfest's theorem

Ehrenfest's theorem is often stated as $\frac{d}{dt}\langle A\rangle = -i\langle[A, H(t)]\rangle$.

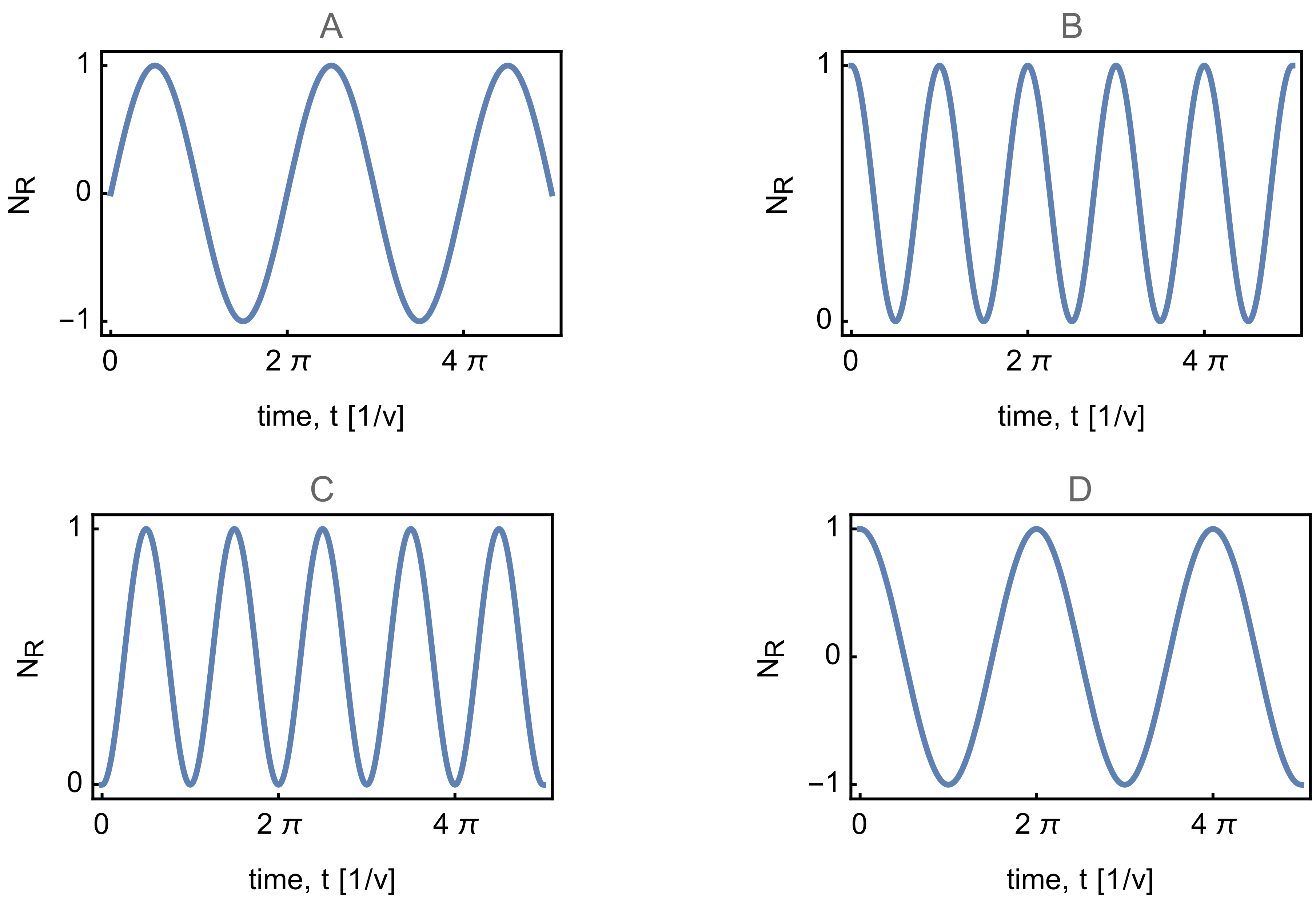
Here, the average $\langle\mathcal{O}\rangle$ corresponds to ...

a) the arithmetic mean of the diagonal elements of the operator $\mathcal{O}$.
b) the expectation value of $\mathcal{O}$ in an arbitrary eigenstate ψ of the Hamiltonian H.
c) the expectation value of $\mathcal{O}$ in an arbitrary state $\psi(t)$.
d) something else.

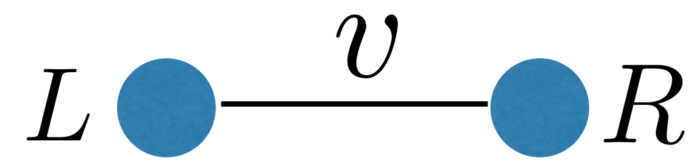
Particle number in a two-site model



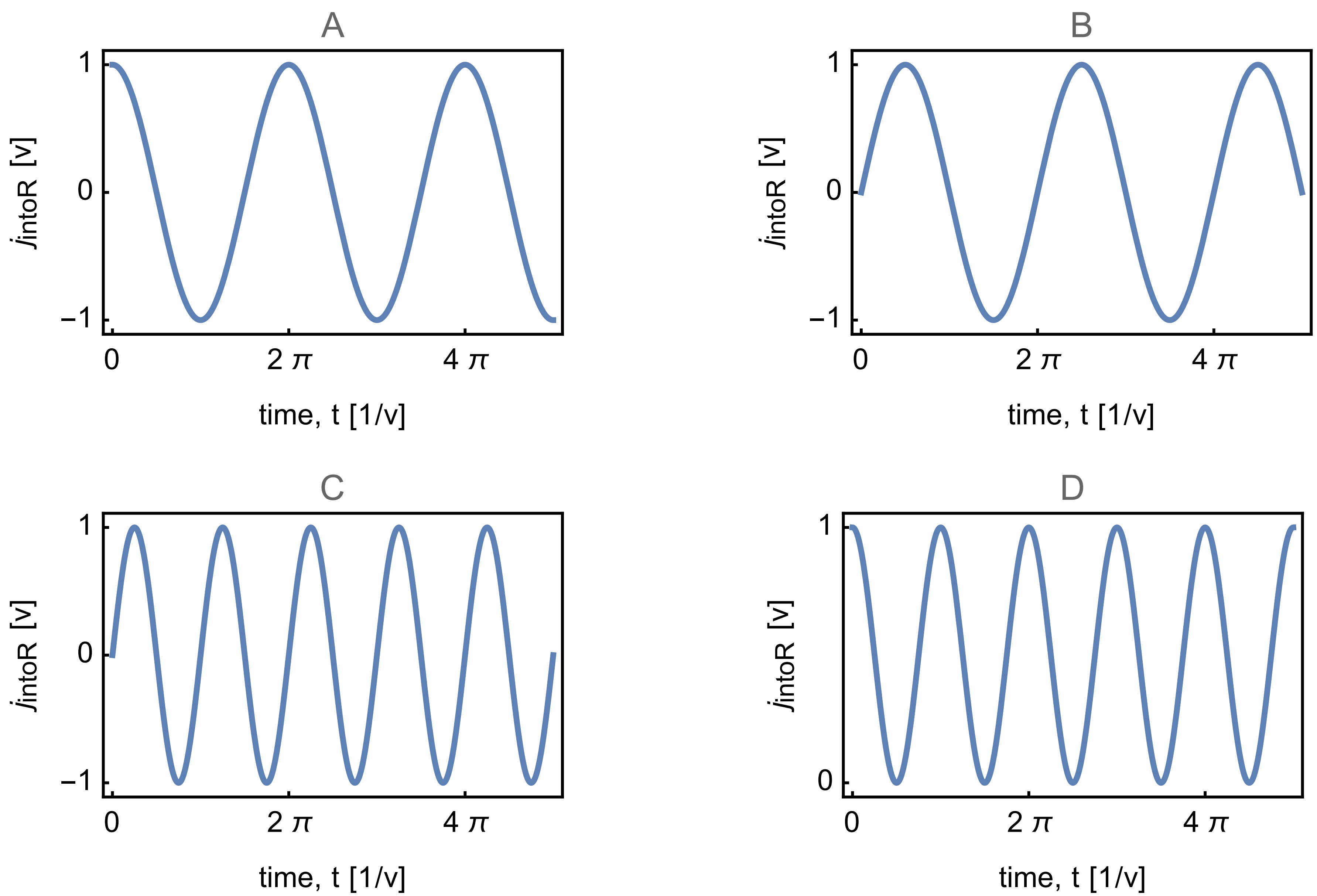
Consider the two-site system described by the Hamiltonian $H = v\sigma_x$. The initial state at $t = 0$ is localized on the left site, $\psi_i(t = 0) = (1, 0)$. How does the particle number $N_R(t)$ on the right site evolve in time?



Current in a two-site model



Consider the two-site system described by the Hamiltonian $H = v\sigma_x$. The initial state at $t = 0$ is localized on the left site, $\psi_i(t = 0) = (1, 0)$. How does the current into the right site, j_{intoR}, evolve in time?



Current in a two-site model II.

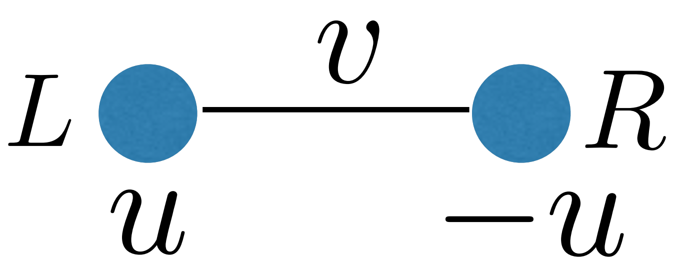
Consider the time-dependent two-site Hamiltonian $H = u(t)\sigma_z + v(t)\sigma_x$. Which of the operators below represents the influx of particles into site R?

a) $-v(t)\sigma_x$

b) $-v(t)\sigma_y$

c) $-iv(t)\sigma_y$

d) $-u(t)\sigma_y$



Particle influx into a segment of a molecule

Consider the 5-atom molecule shown on the right.
The spatial structure of the nonzero hopping amplitudes is indicated by the graph.
Otherwise, hopping amplitudes and on-site energies are arbitrary.

Denote the current operator describing the influx of electrons into the orange segment as $\hat{j}_S$.
The matrix representation of $\hat{j}_S$ in the real-space basis (shown in the figure) is a 5x5 matrix.

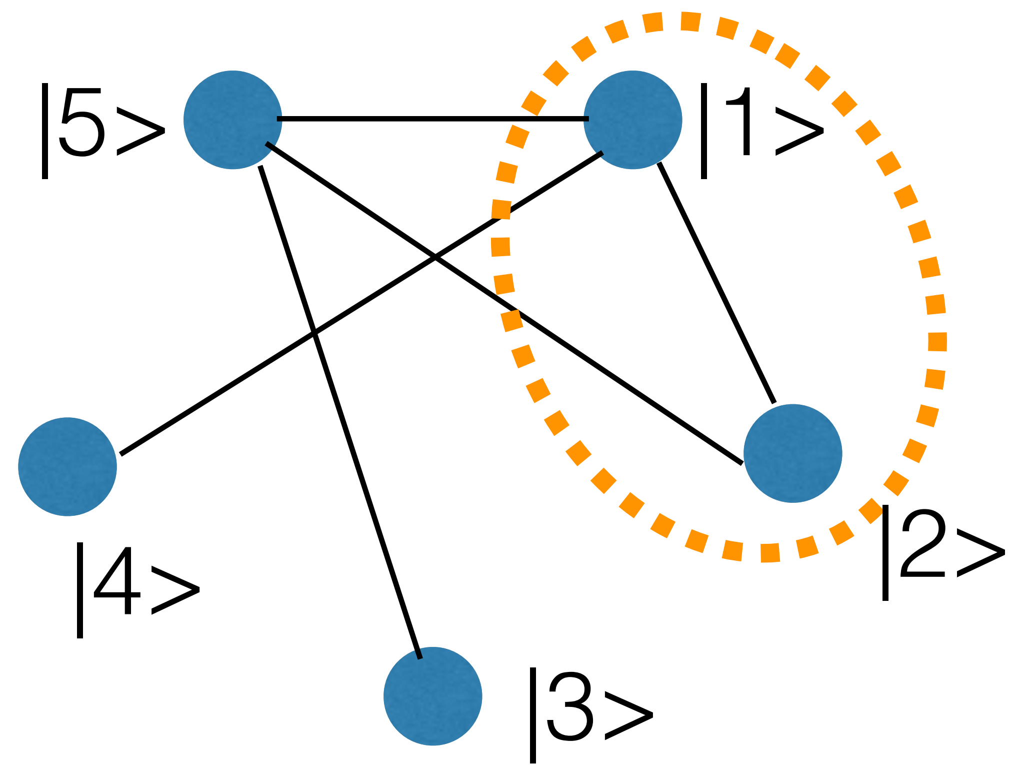
How many nonzero elements does it have?

a) 3

b) 6

c) 8

d) 10



Adiabatic limit of a quasi-adiabatic pumping cycle

Consider the adiabatic limit of a quasi-adiabatic pumping cycle in a 1D crystal.
Which statement is true?

In the adiabatic limit,

a) the momentum- and time-resolved current through a cross section approaches zero.

b) the time-resolved current through a cross section approaches zero.

c) the number of particles pumped through a cross section during the whole cycle approaches zero.

d) More than one of the above statements is true.

Persistent current I.

Take the filled lower-energy band of a static,
insulating one-dimensional, two-band lattice model.
Assume a finite number N of unit cells, periodic boundary condition,
and real-valued hopping amplitudes.

Then,
a) the current carried by each occupied Bloch state is zero.
b) the net current carried by the electrons of the filled band is zero.
c) the net current carried by the electrons of the filled band is always nonzero.
d) the net current carried by the electrons of the filled band can be nonzero.

Persistent current II.

Take the filled lower-energy band of a static, insulating one-dimensional, two-band lattice model. Assume a finite number N of unit cells, periodic boundary condition, but now allow for complex-valued hopping amplitudes.

Then,

a) the current carried by each occupied Bloch state is zero.
b) the net current carried by the electrons of the filled band is zero.
c) the net current carried by the electrons of the filled band is always nonzero.
d) the net current carried by the electrons of the filled band can be nonzero.

Persistent current III.

Take the filled lower-energy band of a static,
insulating one-dimensional, two-band lattice model.
Assume periodic boundary condition, allow for complex-valued hopping amplitudes,
but consider the thermodynamic limit, $N \to \infty$.

Then,
a) the current carried by each occupied Bloch state is zero.
b) the net current carried by the electrons of the filled band is zero.
c) the net current carried by the electrons of the filled band is always nonzero.
d) the net current carried by the electrons of the filled band can be nonzero.

Parallel-transport time parametrization

Consider a spin aligned with a B-field along z.
Adiabatically rotate the B-field 360 degrees in the x-z plane,
such that it returns to its original alignment at the end of the cycle:

$H(t) = \boldsymbol{B}(t) \cdot \boldsymbol{\sigma}$, where $\boldsymbol{B}(t) = B(\sin(2\pi t/T), 0, \cos(2\pi t/T))$.

Let us describe the instantaneous ground state of
this Hamiltonian with the parallel-transport
time parametrization that starts with $\psi(t=0) = (0,1)$.

What is the value of this parametrization in the final point $t = T$?

a) $\psi(T) = (0,1)$

b) $\psi(T) = -(0,1)$

c) $\psi(T) = e^{iBT}(0,1)$

d) $\psi(T) = -e^{iBT}(0,1)$