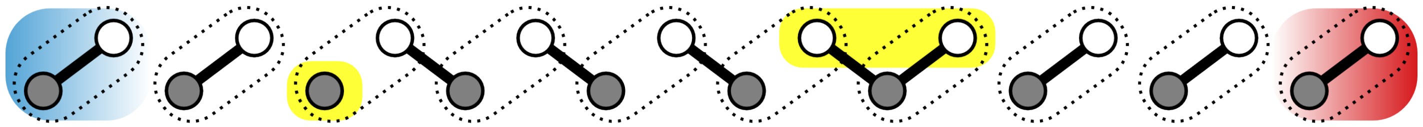
Chapter 7

Continuum model of localized states at a domain wall

2017. 10. 10

Earlier: 1D SSH model 

bulk / half-infinite / finite / **domain wall**

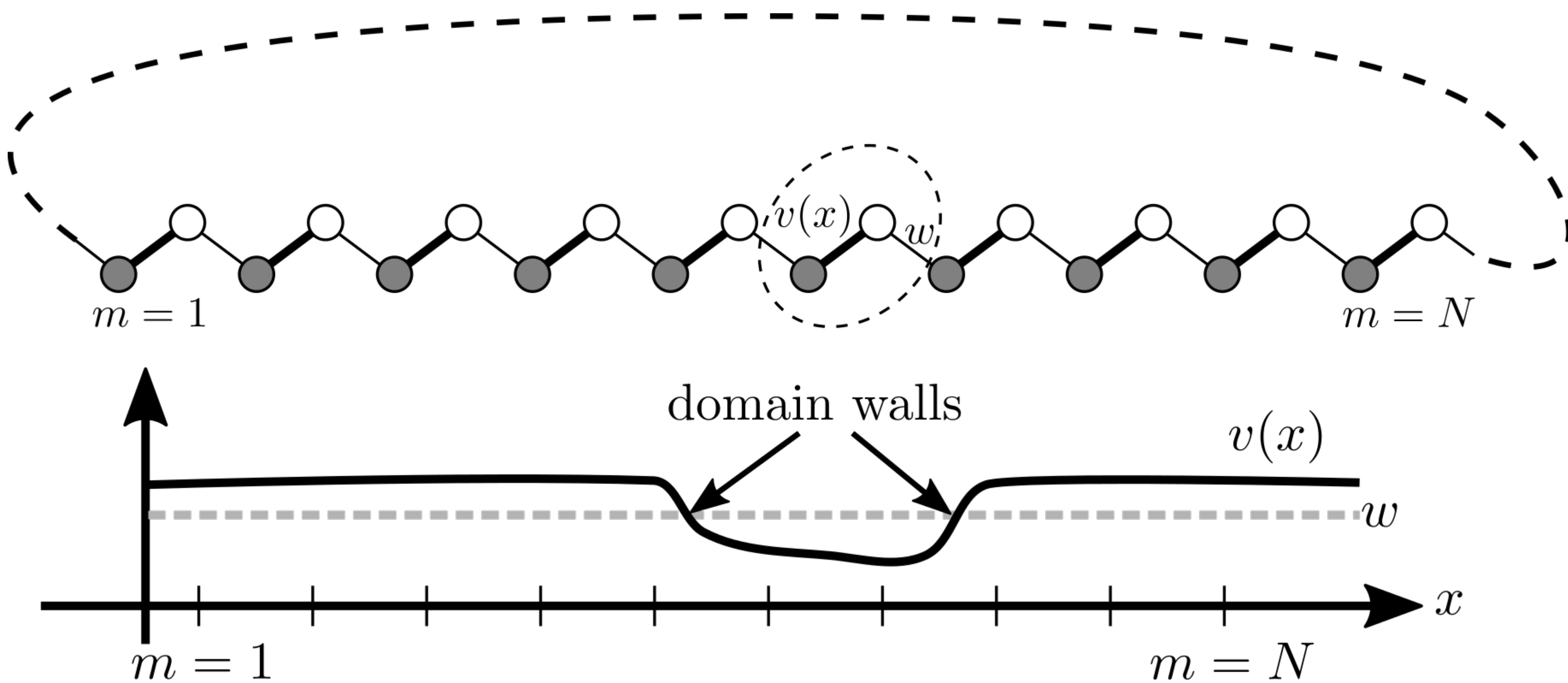
zero-energy states

Now: lattice => continuum

matrix Hamiltonian => differential operator Hamiltonian

Recipe to derive the continuum model

example:
SSH model,
inhomogeneous
intracell hopping

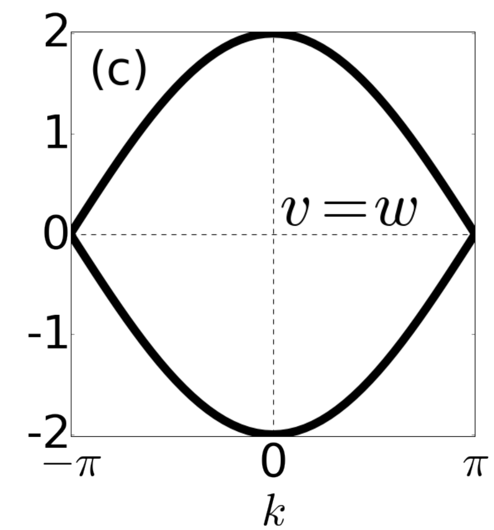


Recipe to derive the continuum model

(1) take bulk momentum-space hamiltonian

$$H(k) = d_x(k)\hat{\sigma}_x + d_y(k)\hat{\sigma}_y + d_z(k)\hat{\sigma}_z$$

$$d_x(k) = v + w\cos k; \qquad d_y(k) = w\sin k; \qquad d_z(k) = 0.$$



(2) identify metallic band crossing (k0 = pi)

(3) assume nearly metallic condition

$$|v(x) - w| \ll v(x) + w$$

$$M = v - w$$

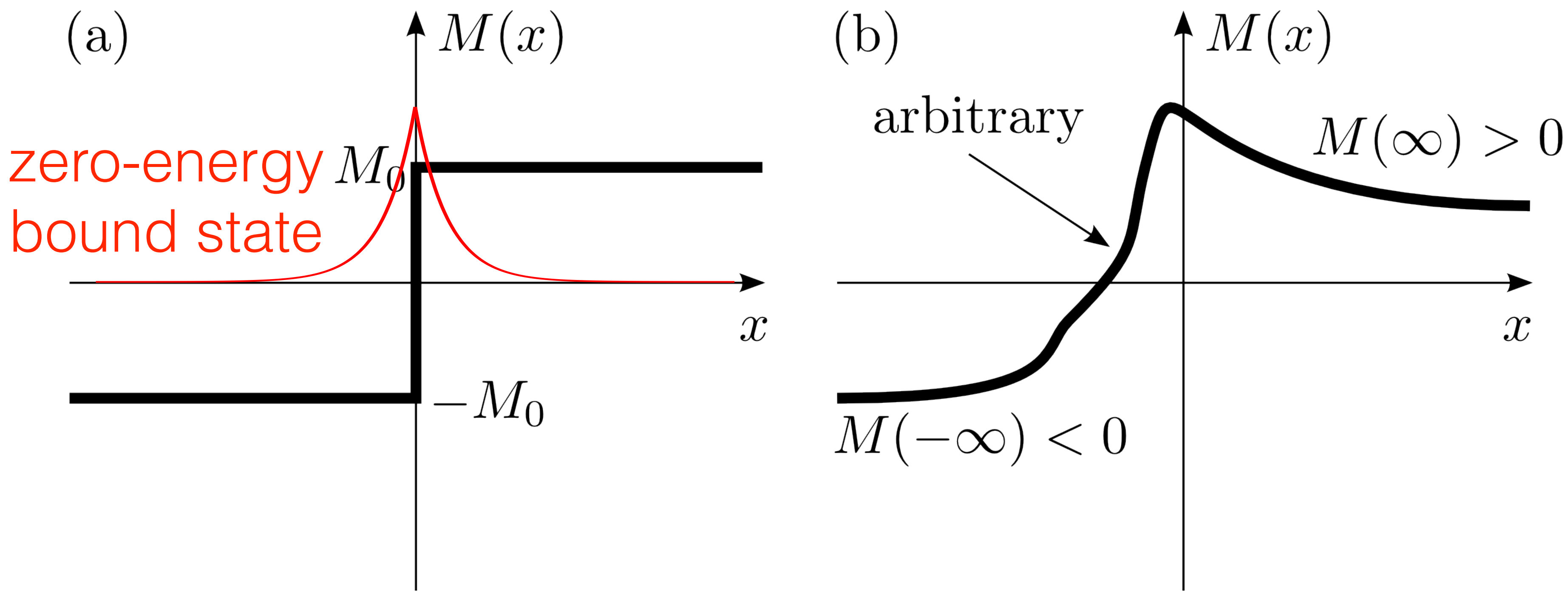
(4) expand H in q=k-k0

(5) insert spatial dependence of parameters & replace q with momentum operator p

$$H_{\text{EFA}} = M(x)\hat{\sigma}_x - w\hat{p}\hat{\sigma}_y.$$

Continuum model describes zero-energy bound state

setup: `mass domain wall' in the 1D SSH model / 1D massive Dirac equation



Continuum model is good

useful: lattice <=> numerics; continuum <=> analytics & numerics

examples:
zero- and finite-energy bound states
electron scattering and electric transport

interesting: Dirac equation => connection to relativistic physics

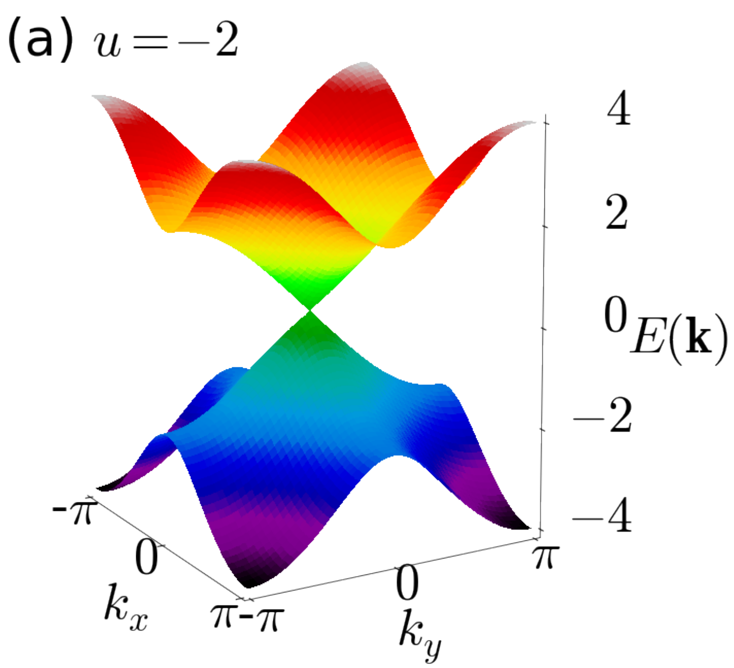
Continuum model is limited

limited to the nearly metallic case
(e.g., does not capture the fully dimerized limit of the SSH model)

limited to a finite range in momentum and energy
(`low-energy continuum model')

Continuum model works in two dimensions as well

setup: `mass domain wall’ in the 2D QWZ model / 2D massive Dirac equation



$$\hat{H}(k) = \sin k_x \hat{\sigma}_x + \sin k_y \hat{\sigma}_y + [u + \cos k_x + \cos k_y]\hat{\sigma}_z.$$

$$\begin{aligned} u < -2 \quad &: \quad Q = 0; \\ -2 < u < 0 \quad &: \quad Q = -1; \end{aligned}$$

$$\hat{H}_{\text{EFA}} = M(x,y)\hat{\sigma}_z + \hat{p}_x \hat{\sigma}_x + \hat{p}_y \hat{\sigma}_y.$$

$$M = u + 2.$$