Chapter 7

Continuum model of localized states at a domain wall

2017. 10. 10

Earlier: 1D SSH model bulk / half-infinite / finite / domain wall zero-energy states

Now: lattice => continuum matrix Hamiltonian => differential operator Hamiltonian
Recipe to derive the continuum model

example:
SSH model,
inhomogeneous
intracell hopping
Recipe to derive the continuum model

1) take bulk momentum-space hamiltonian

\[ H(k) = d_x(k)\hat{\sigma}_x + d_y(k)\hat{\sigma}_y + d_z(k)\hat{\sigma}_z \]

\[ d_x(k) = v + w\cos k; \quad d_y(k) = w\sin k; \quad d_z(k) = 0. \]

2) identify metallic band crossing \((k0 = \pi)\)

3) assume nearly metallic condition \( |v(x) - w| \ll v(x) + w \)

\[ M = v - w \]

4) expand \( H \) in \( q = k - k0 \)

5) insert spatial dependence of parameters & replace \( q \) with momentum operator \( p \)

\[ H_{\text{EFA}} = M(x)\hat{\sigma}_x - w\hat{p}\hat{\sigma}_y. \]
Continuum model describes zero-energy bound state

setup: `mass domain wall' in the 1D SSH model / 1D massive Dirac equation
Continuum model is good

**useful**: lattice $\iff$ numerics; continuum $\iff$ analytics & numerics

examples:
- zero- and finite-energy bound states
- electron scattering and electric transport

**interesting**: Dirac equation $\Rightarrow$ connection to relativistic physics
Continuum model is limited

limited to the nearly metallic case
(e.g., does not capture the fully dimerized limit of the SSH model)

limited to a finite range in momentum and energy
(`low-energy continuum model’)
Continuum model works in two dimensions as well

setup: `mass domain wall` in the 2D QWZ model / 2D massive Dirac equation

\( \hat{H}(k) = \sin k_x \hat{\sigma}_x + \sin k_y \hat{\sigma}_y + [u + \cos k_x + \cos k_y] \hat{\sigma}_z. \)

\( u < -2 \quad : \quad Q = 0; \)
\( -2 < u < 0 \quad : \quad Q = -1; \)

\( \hat{H}_{EFA} = M(x, y) \hat{\sigma}_z + \hat{p}_x \hat{\sigma}_x + \hat{p}_y \hat{\sigma}_y. \)

\( M = u + 2. \)