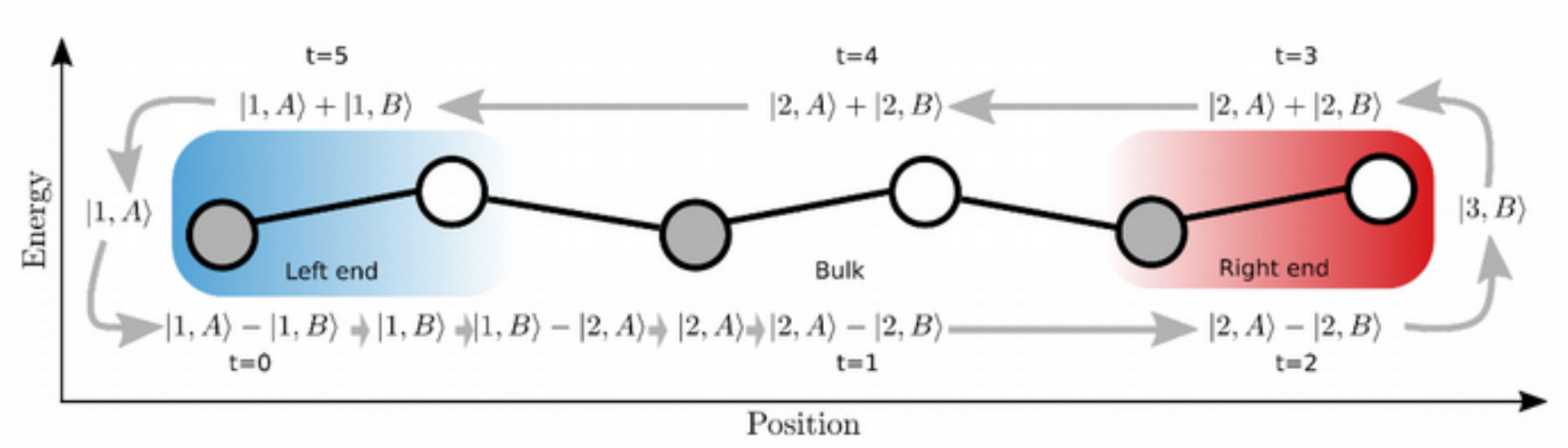
6. Chern Insulators: The Qi-Wu-Zhang Model

Most important chapter: heart of topological insulators.

- Required: Thouless pumping
- New theory tool: Promoting time $t \rightarrow$ quasimomentum k
- Main results: Edge states in two-dimensional systems
  Bulk Chern number predicts edge states
  Topological protection

- Toy model: Qi-Wu-Zhang (obtained from Thouless pump in Rice-Mele by promoting $t \rightarrow k$

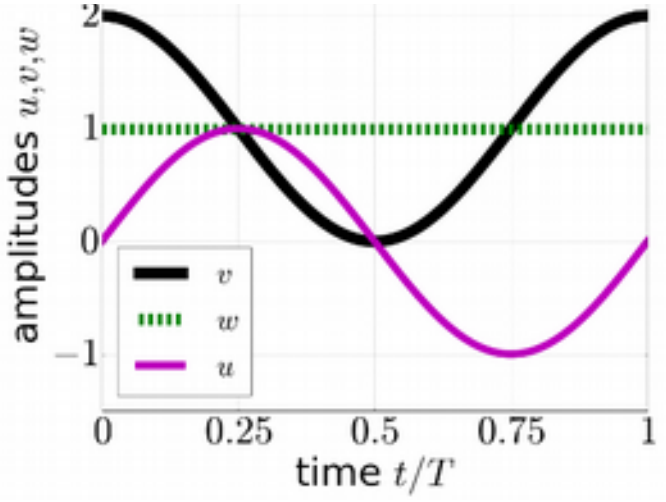
Reminder 1: Thouless pump sequence, Rice-Mele

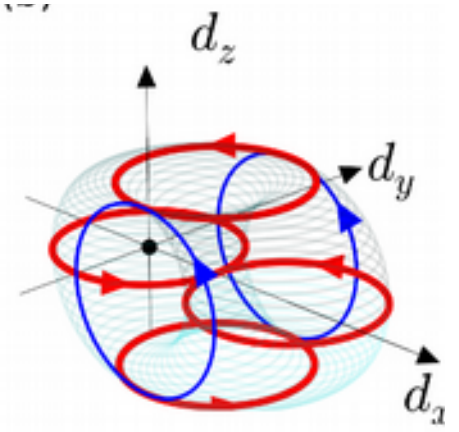


Pump charge along a dimerized chain using sublattice potential:

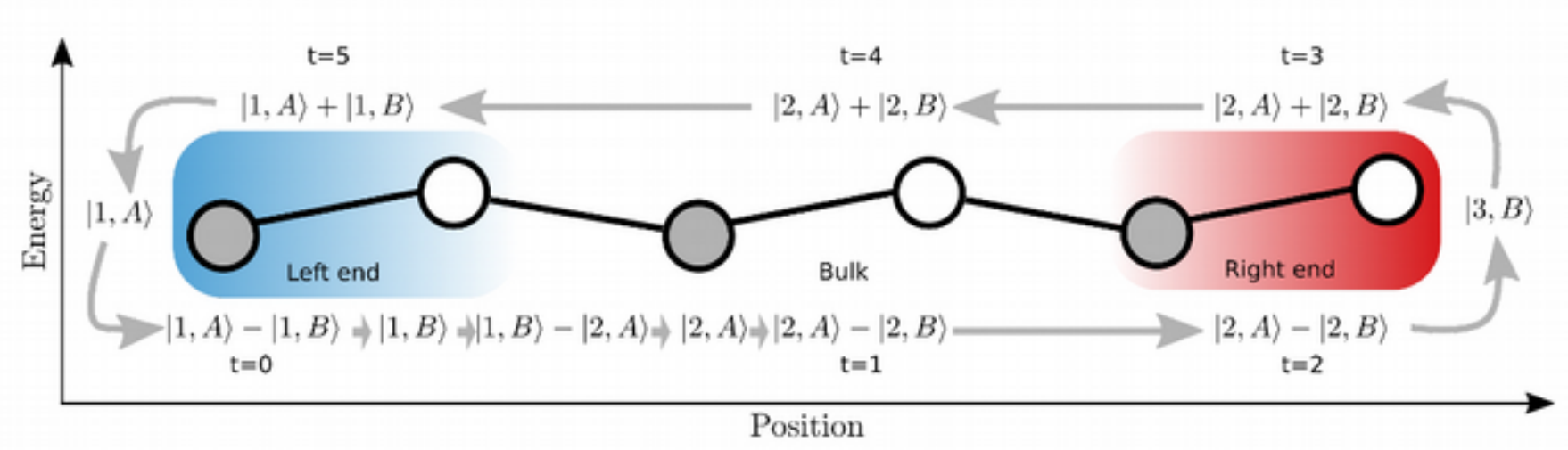
$$\hat{H}(k,t) = \mathbf{d}(k,t)\hat{\sigma} = (v(t) + w(t)\cos k)\hat{\sigma}_x + w(t)\sin k\hat{\sigma}_y + u(t)\hat{\sigma}_z$$

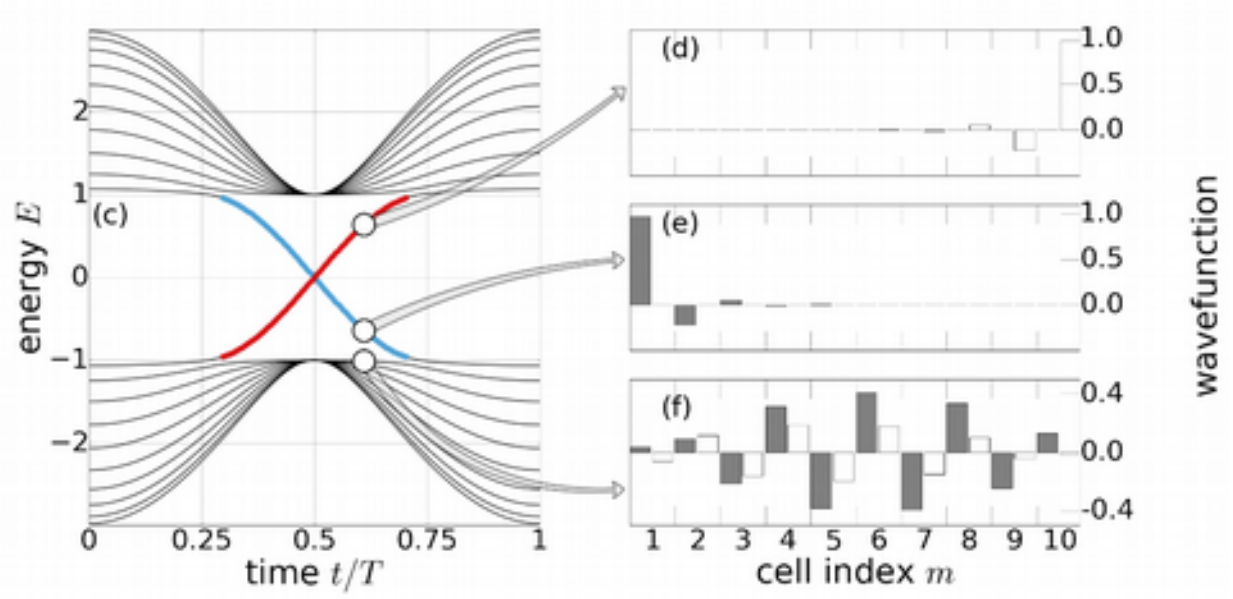
$$u(t) = \sin\Omega t;$$
$$v(t) = \overline{v} + \cos\Omega t;$$
$$w(t) = 1,$$





Reminder 2: Topologically Protected Edge States in Thouless pump

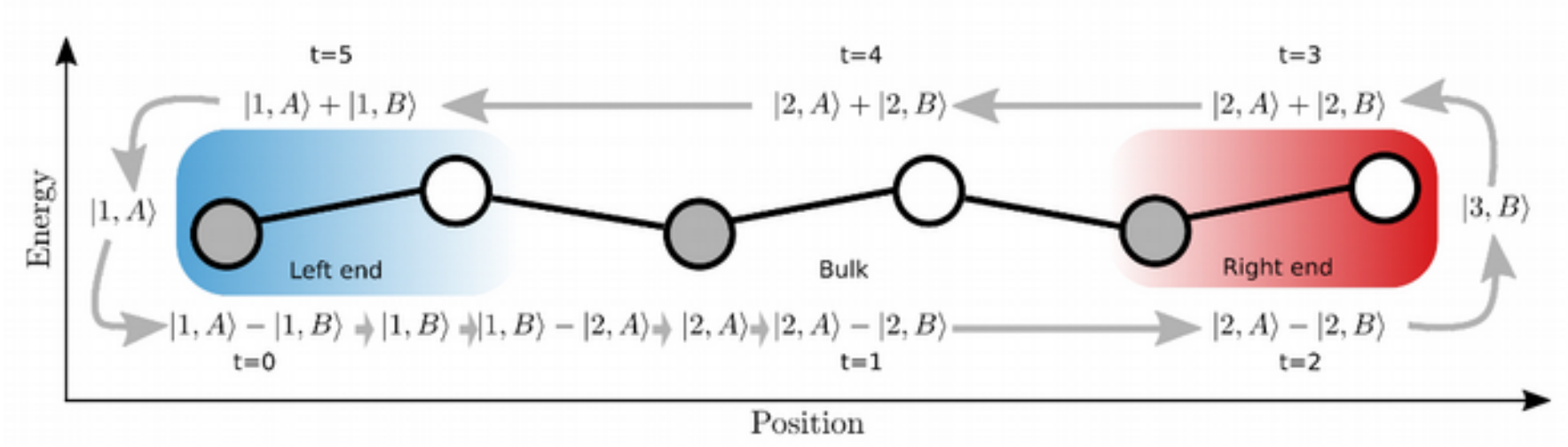




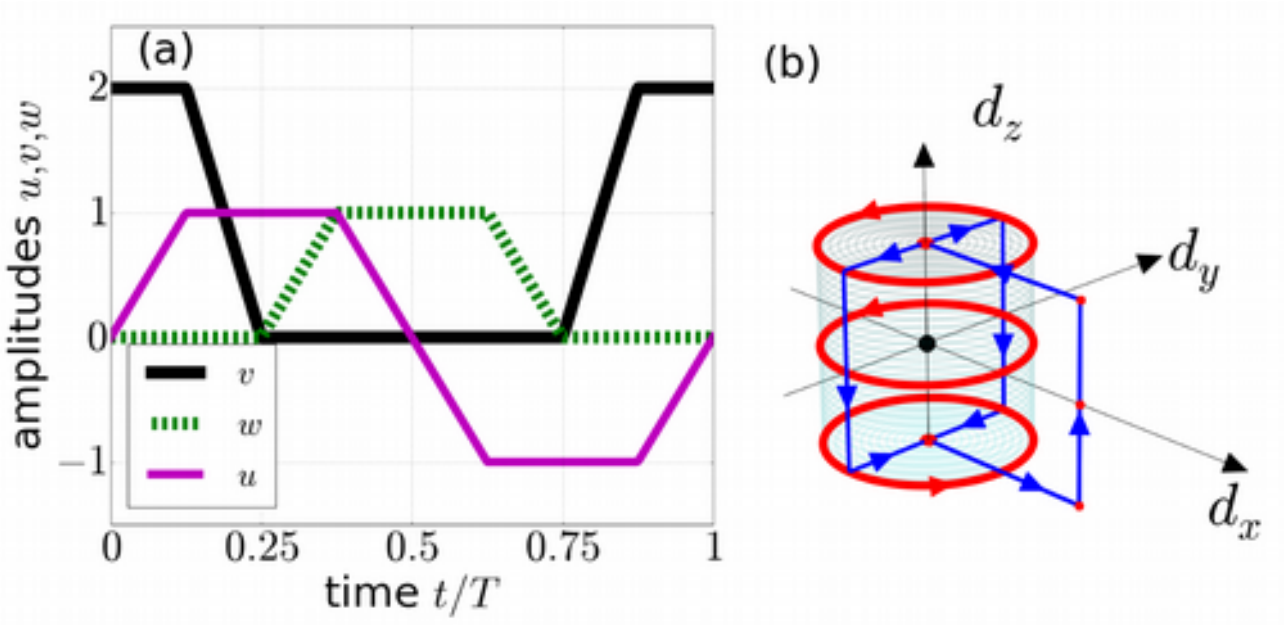
Topologically protected = robust:
1) Time - Periodic drive
2) No long range hopping

1→ spectrum time-periodic
2→ spectrum continuous
2→ bulk gap separates two edges
→ no direct coupling,
→ crossing, not anticrossing

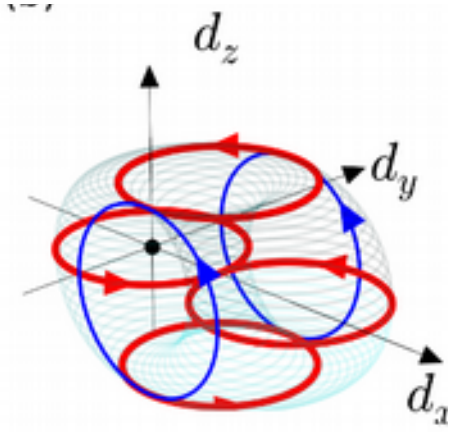
Reminder 3: Thouless pump in the bulk in d-space: # times origin in torus = # charge pumped = Chern



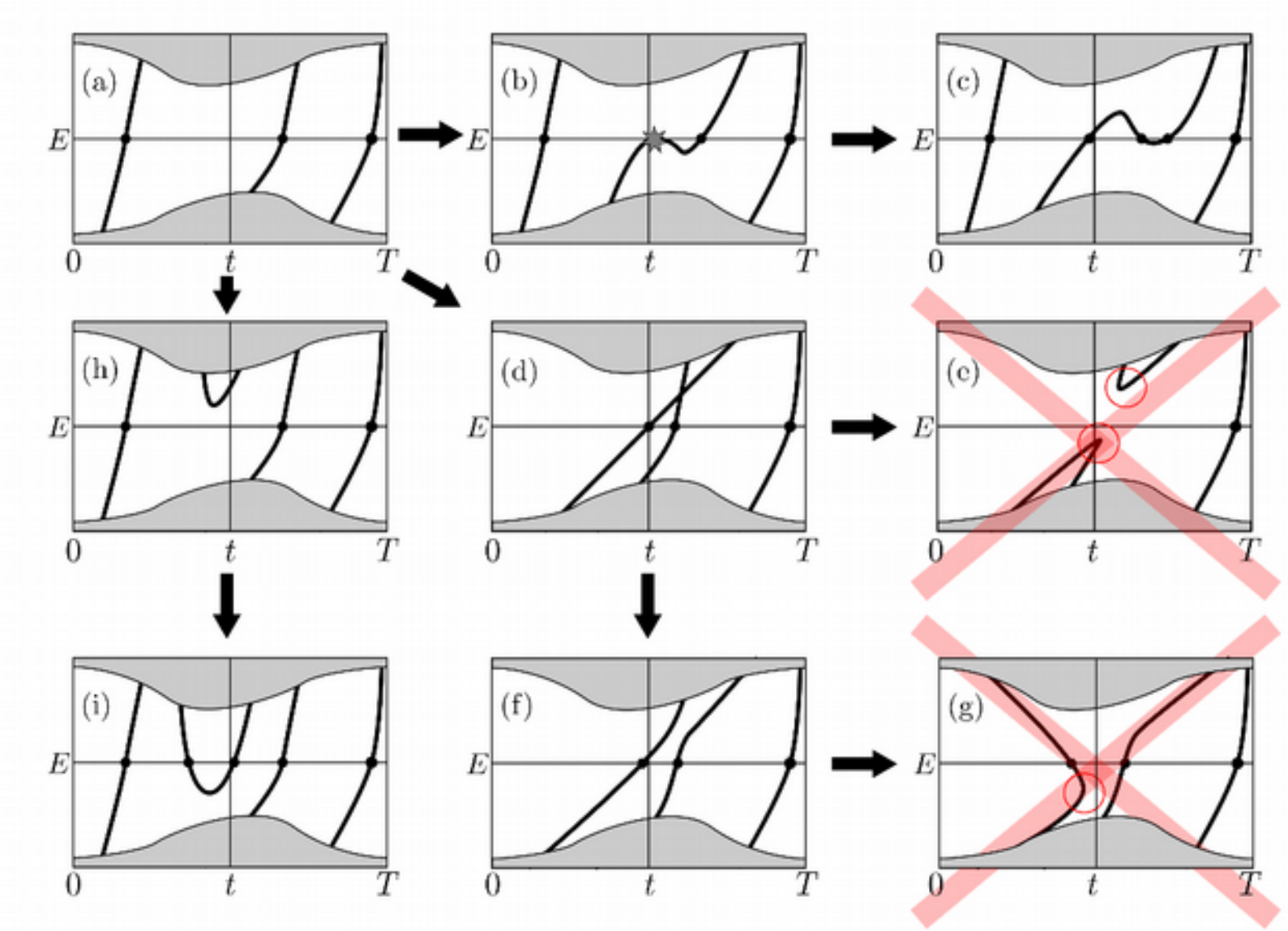
control freak sequence:



smooth sequence:



Reminder 4: Net number of charge pumped up in energy at an edge is protected against continuous deformations



New material, class 6:
From Thouless pump to Chern insulator

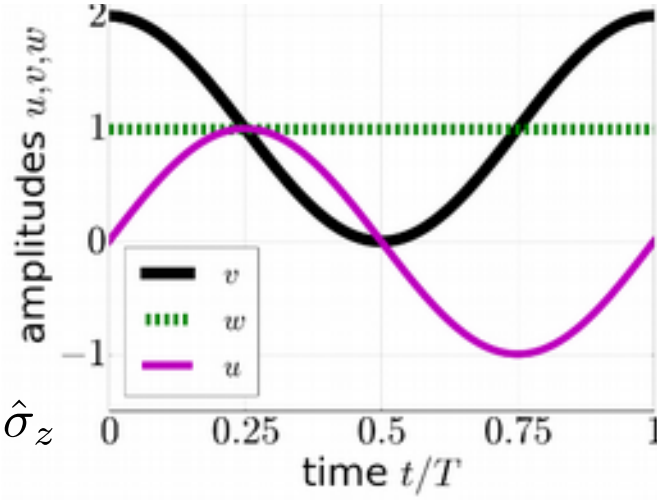
Promote time $t \rightarrow$ wavenumber k
1D time-periodic Rice-Mele $\rightarrow$ 2D Qi-Wu-Zhang

$$\hat{H}_{\mathrm{RM}}(k,t) = \sin k\hat{\sigma}_y + \sin \Omega t \hat{\sigma}_z + (\bar{v} + \cos \Omega t + \cos k)\hat{\sigma}_x$$

$\Omega t \rightarrow k_y$ $\quad$ $\hat{\sigma}_y \rightarrow \hat{\sigma}_x$ $\quad$ $\bar{v} \rightarrow u$

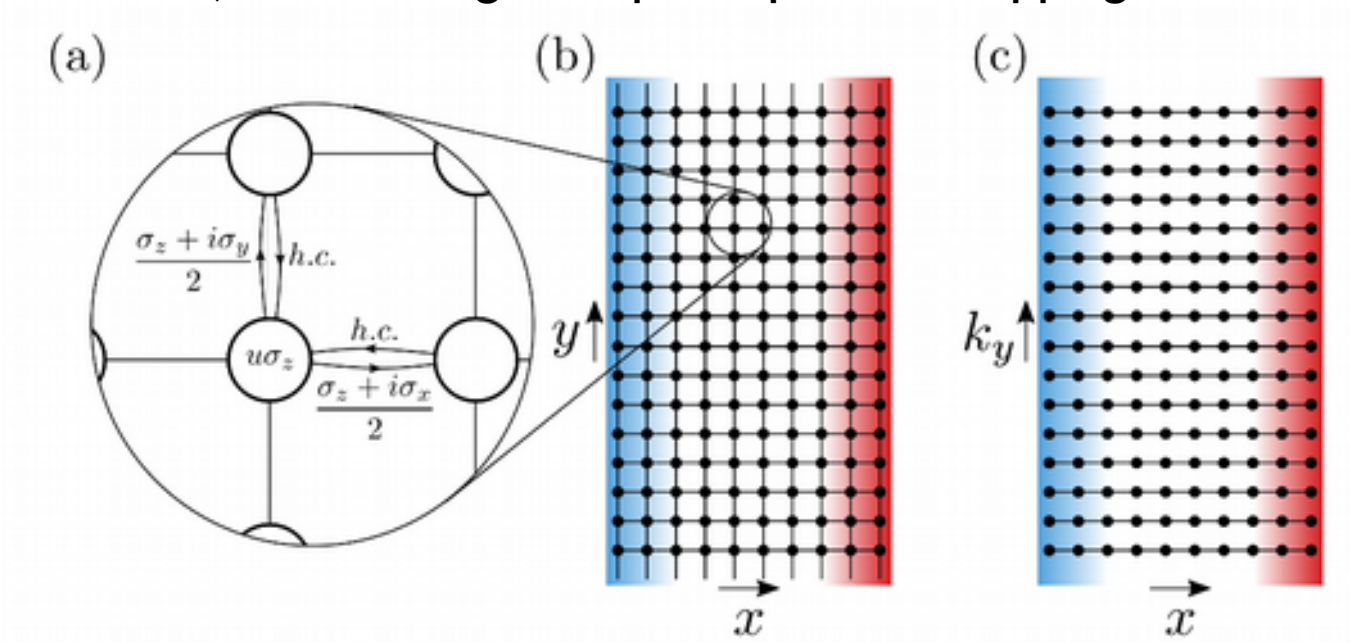
$k \rightarrow k_x$ $\quad$ $\hat{\sigma}_z \rightarrow \hat{\sigma}_y$

$\hat{\sigma}_x \rightarrow \hat{\sigma}_z$

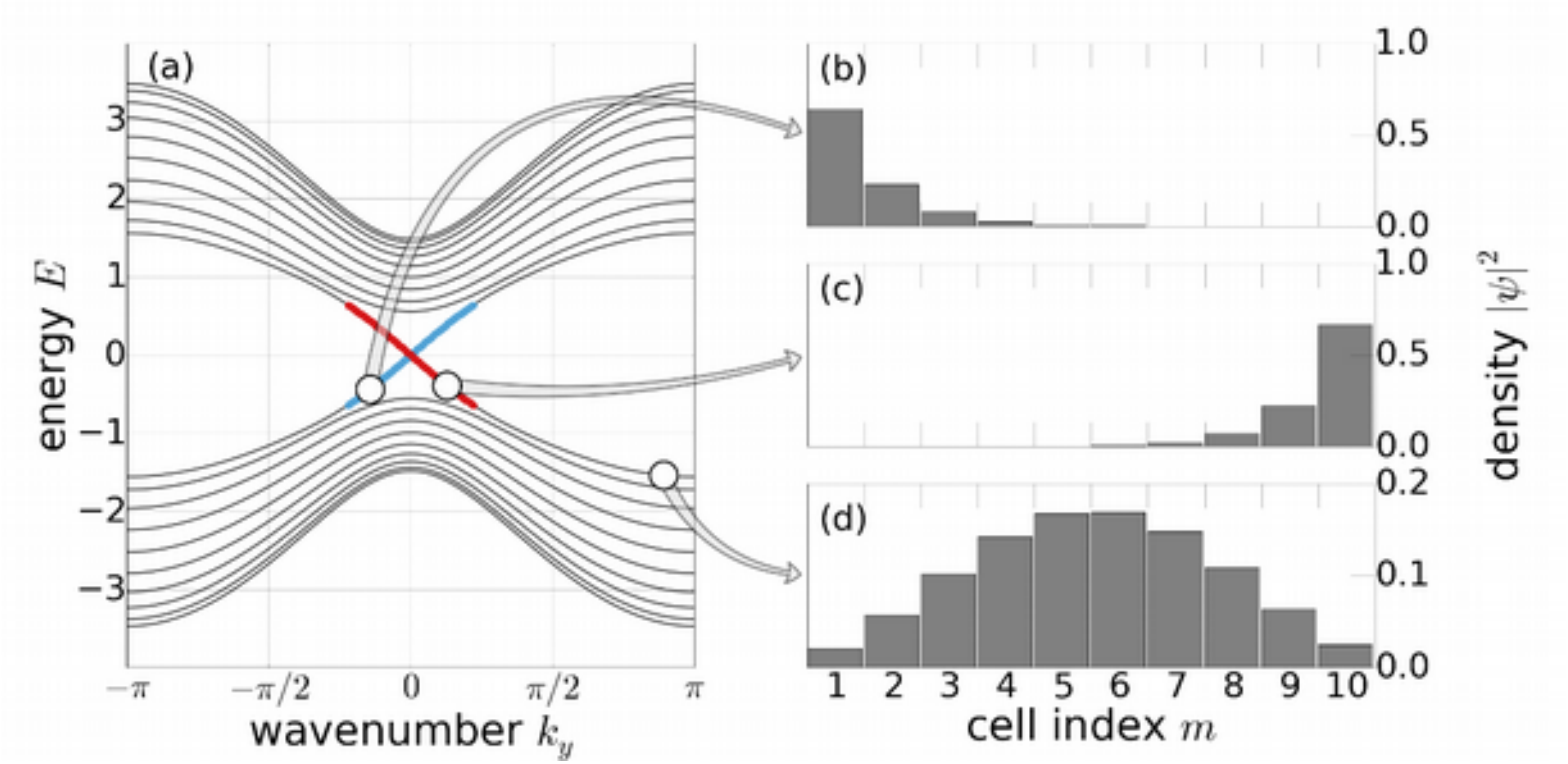


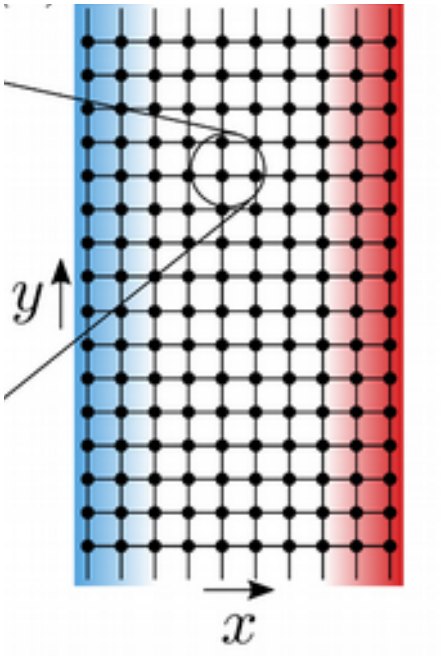
$$\hat{H}_{\mathrm{QWZ}}(k_x,k_y) = \sin k_x\hat{\sigma}_x + \sin k_y\hat{\sigma}_y + (u + \cos k_x + \cos k_y)\hat{\sigma}_z$$

2D square lattice, nearest-neighbor spin-dependent hopping



Edge states rising/falling in Thouless pump
→ unidirectional edge modes in Chern insulators

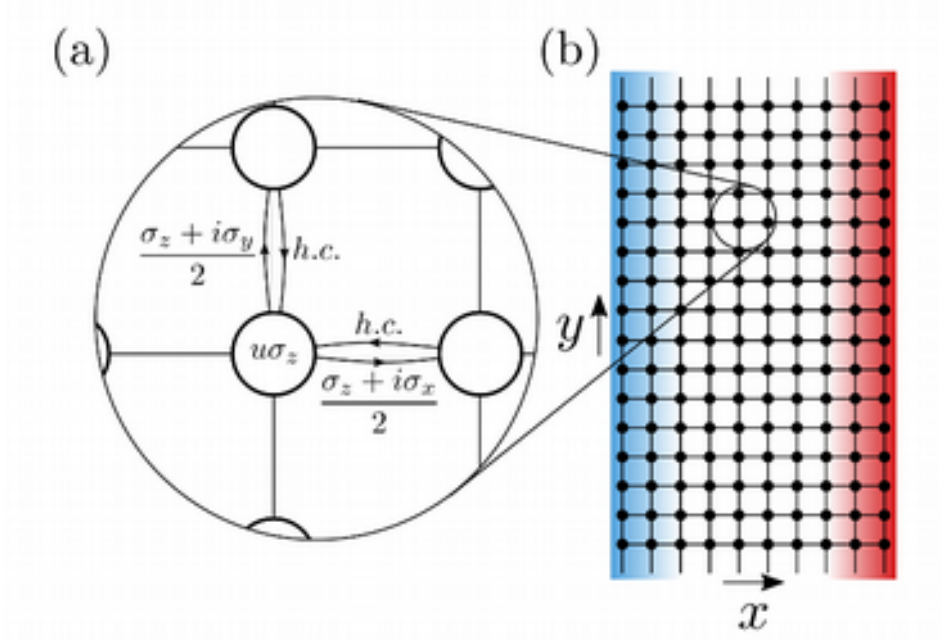


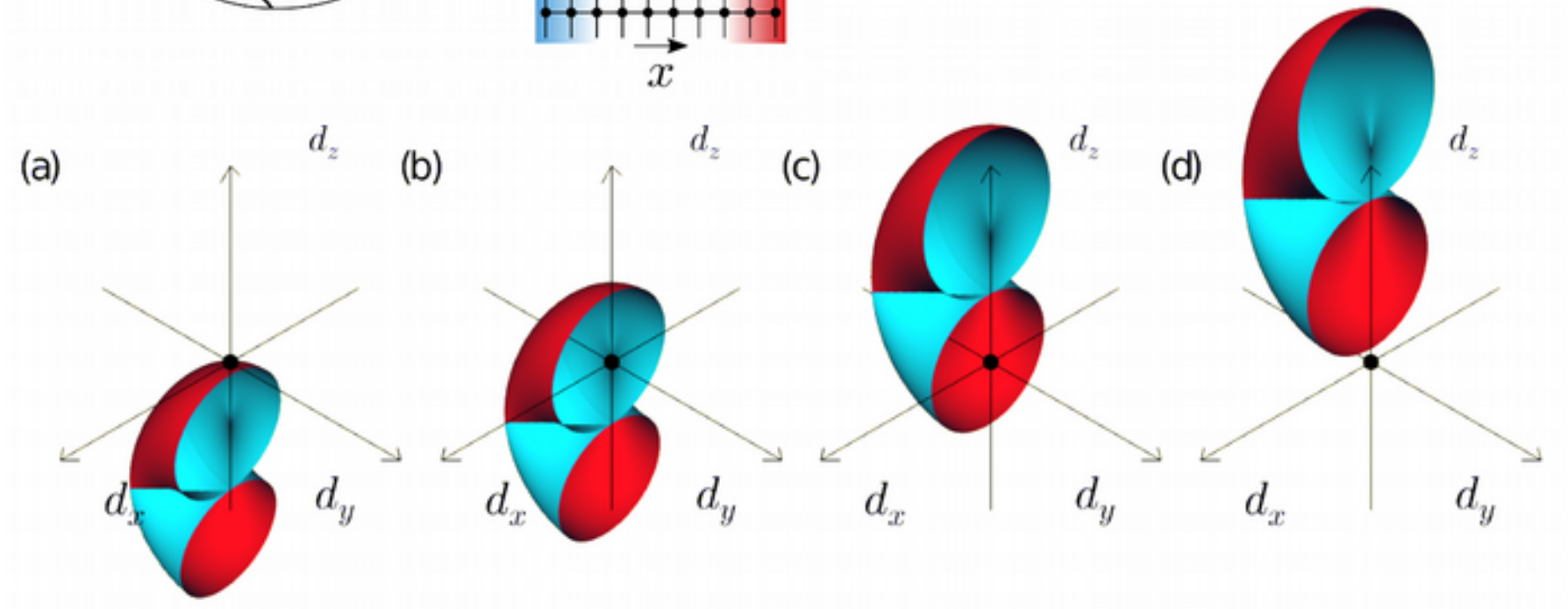


Topologically protected = robust:
- No long range hopping

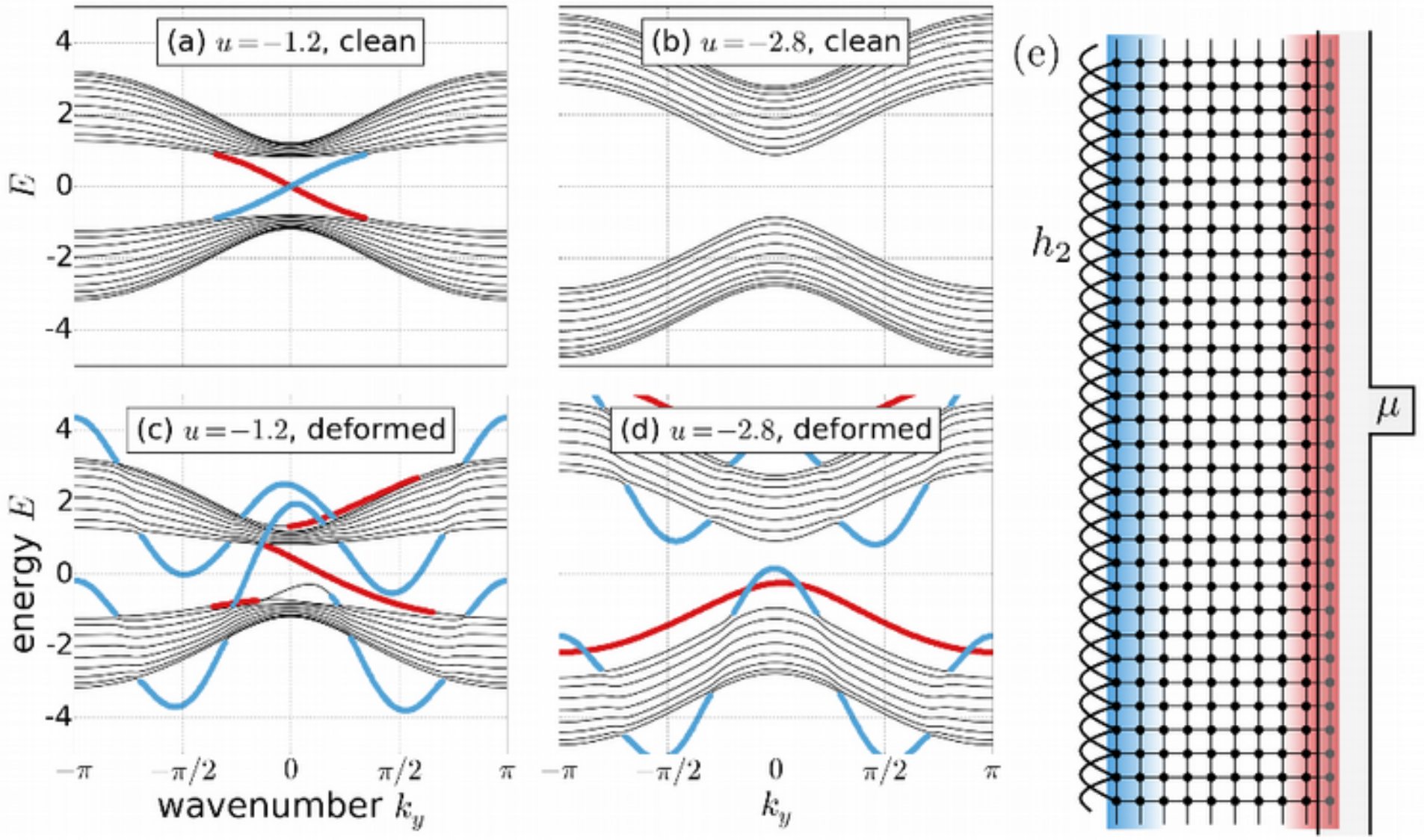
→ spectrum periodic & smooth
→ bulk gap separates two edges → no direct coupling → crossing, not anticrossing

Presence, net # of edge state modes seen in bulk:
times origin in torus = # edge state modes = Chern

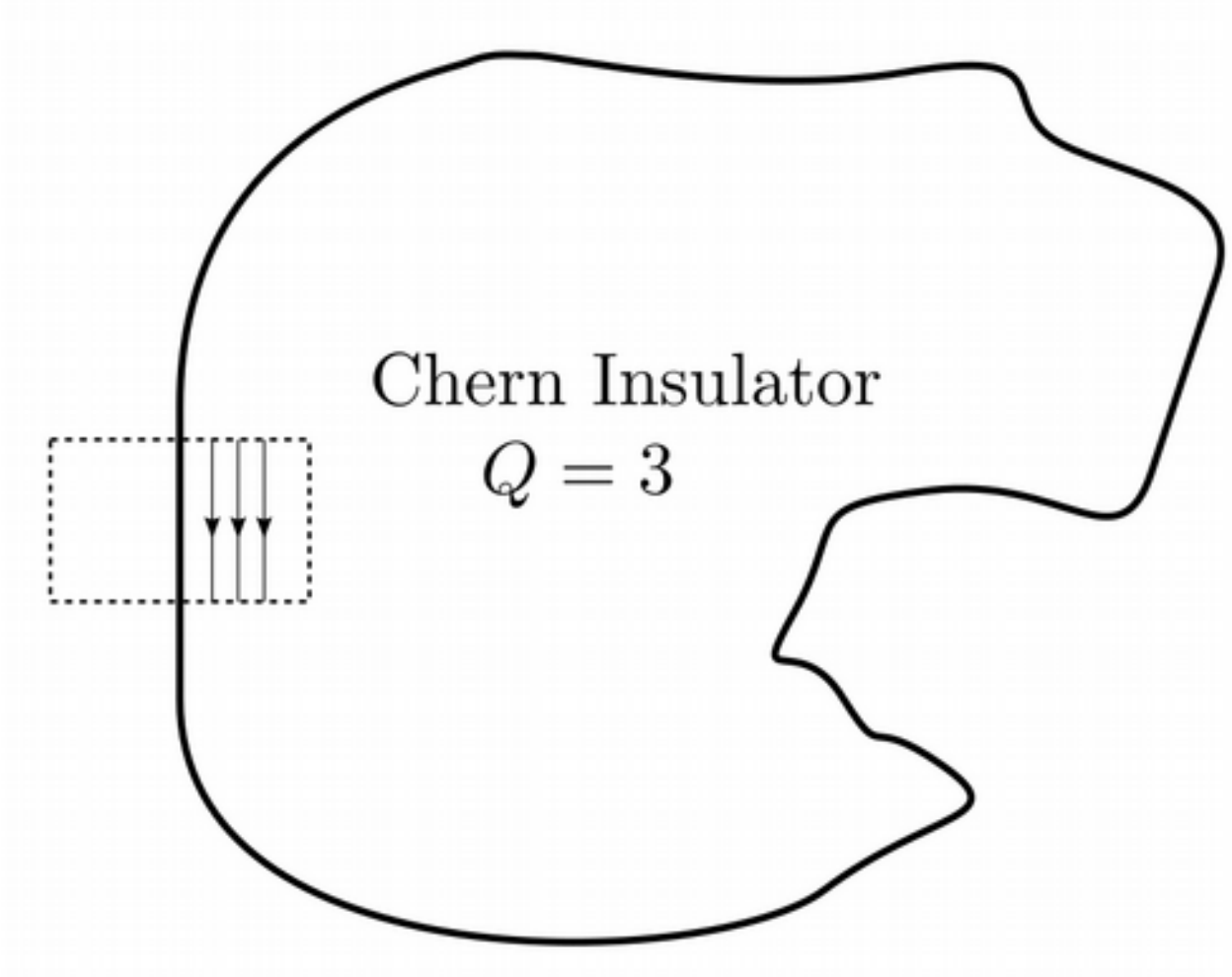




Net number of clockwise-propagating edge state modes in the gap is protected against continuous deformations



Net edge states at some section of edge → edge states all around (unitarity → particles cannot accumulate)



Topologically protected = robust against:

- Arbitrary disorder on edges
- Some disorder in bulk
  (interesting variation on Anderson localization)

Summary: Chern Insulators have robust edge states predicted by bulk Chern

- Required: Thouless pumping (ensure edge states, Chern #)
- Theory tool: Promote time $t \rightarrow$ quasimomentum k
- Main results: Edge states in two-dimensional systems
  Bulk Chern number predicts edge states
  Topological protection due to no backscattering
  Robust against disorder (large edge, small bulk)
- Toy model: Qi-Wu-Zhang (from Thouless pump Rice-Mele)
  Tune Chern number by
  onsite magnetic field u (-2, 0, 2)

$$\hat{H}_{\mathrm{QWZ}}(k_x, k_y) = \sin k_x \hat{\sigma}_x + \sin k_y \hat{\sigma}_y + (u + \cos k_x + \cos k_y)\hat{\sigma}_z$$

Budapest, 2017 November 3