Experimental realization
of
Majorana fermions

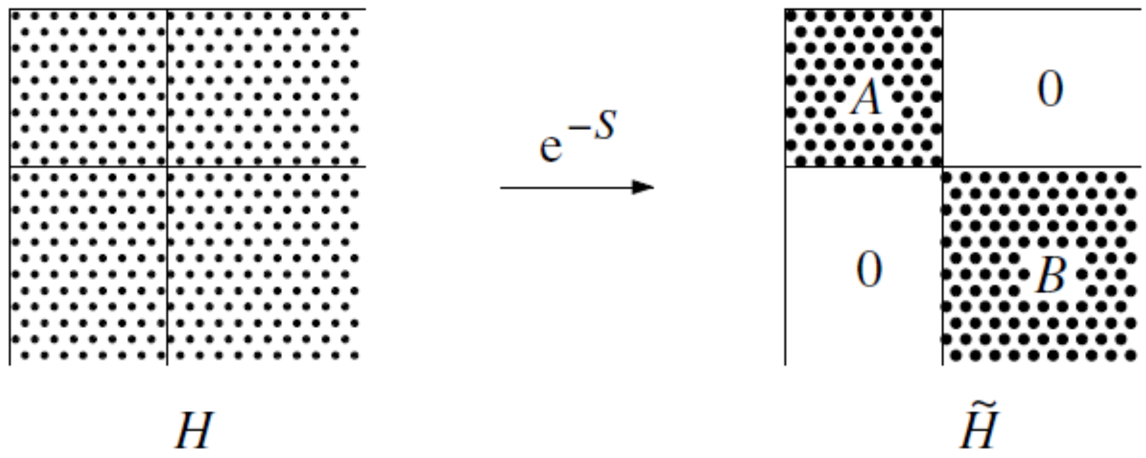
-Quasi degenerate perturbation theory

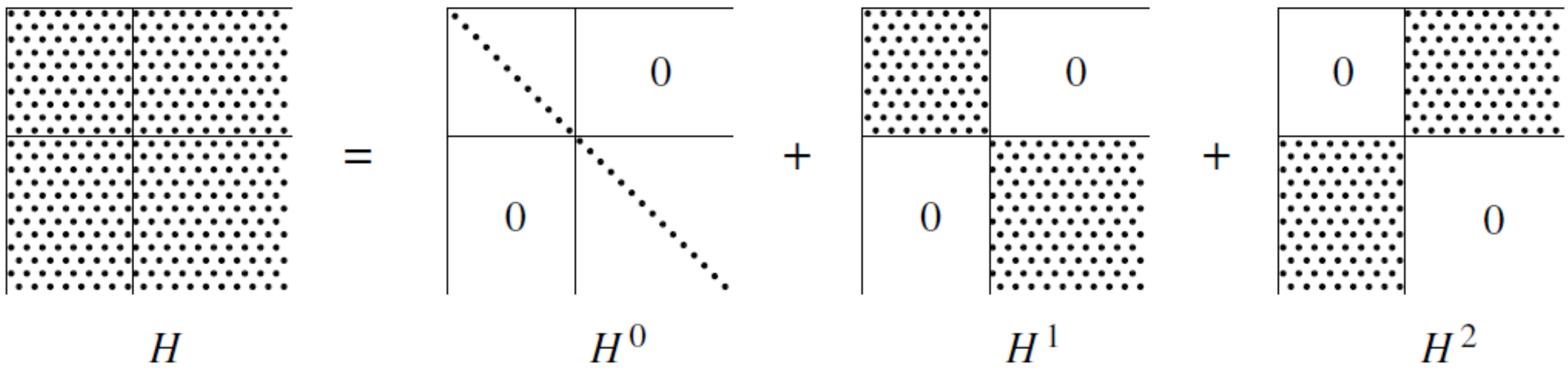
-Proximity effect near s-wave

-Effective p-wave pairing from s-wave
proposals for experiments

-Delft experiment

Quasi degenerate perturbation theory





Quasi degenerate perturbation theory

$$H = H^0 + H' = H^0 + H^1 + H^2$$

$$\tilde{H} = \mathrm{e}^{-S} H \mathrm{e}^{S} = \sum_j \frac{1}{j!} [H, S]^{(j)}$$

H^0 diagonal part of diagonal block

H^1 off-diagonal part of diagonal block

H^2 off-diagonal block

S choose off-diagonal block!

$$\tilde{H}_d = \sum_j \frac{1}{(2j)!} \left[H^0 + H^1, S\right]^{(2j)} + \sum_j \frac{1}{(2j+1)!} \left[H^2, S\right]^{(2j+1)}$$

$$\tilde{H}_n = \sum_j \frac{1}{(2j+1)!} \left[H^0 + H^1, S\right]^{(2j+1)} + \sum_j \frac{1}{(2j)!} \left[H^2, S\right]^{(2j)} =! = 0$$

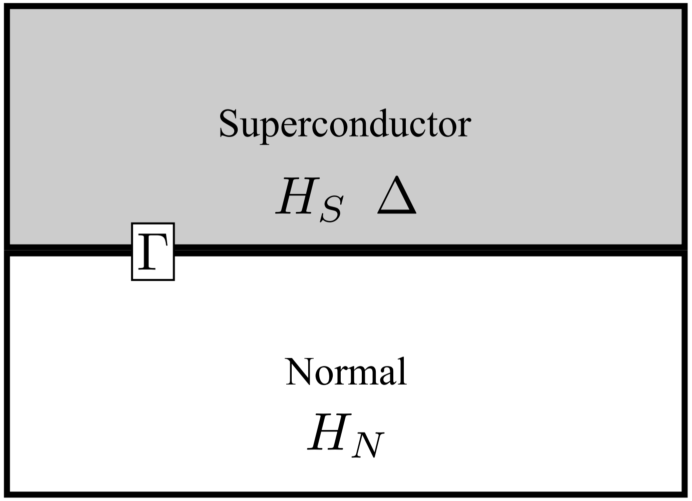
Results up to second order in H':

$$H^{(0)}_{mm'} = H^0_{mm'}$$

$$H^{(1)}_{mm'} = H'_{mm'}$$

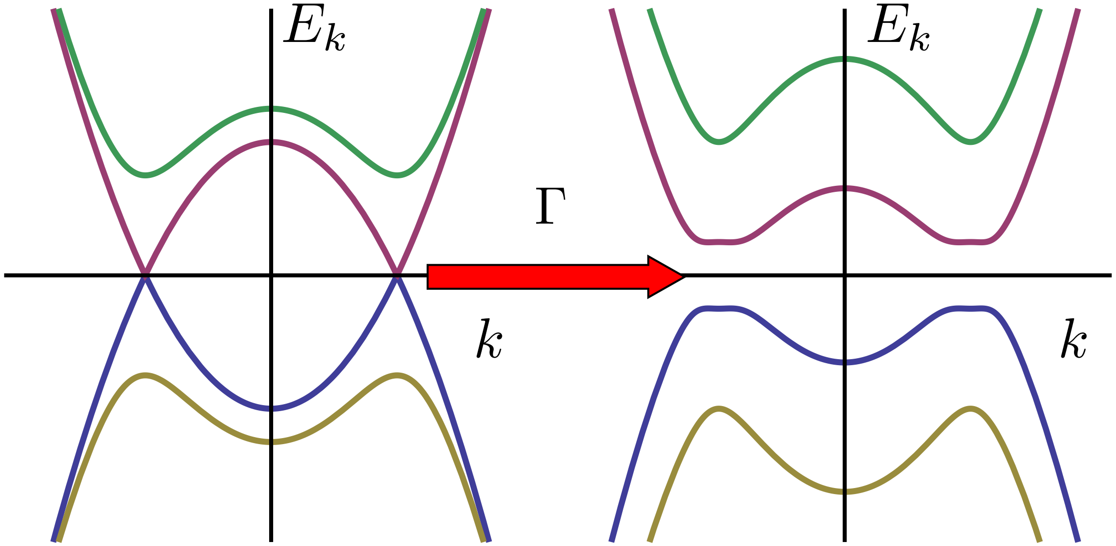
$$H^{(2)}_{mm'} = \frac{1}{2} \sum_l \frac{H'_{ml}}{E_m - E_l} H'_{lm'} + H'_{ml} \frac{H'_{lm'}}{E_{m'} - E_l}$$

Proximity effect



$$H = \sum_{ij} h^N_{ij} c^\dagger_i c_j + \sum_{ij} h^S_{ij} b^\dagger_i b_j + \sum_{ij} (\Delta_{ij} b_i b_j + h.c.) + \sum_{ij} \left(\Gamma_{ij} b^\dagger_i c_j + h.c.\right)$$

$$H_{BdG} = \begin{pmatrix} H_N & 0 & \Gamma & 0 \\ 0 & -H_N^* & 0 & -\Gamma \\ \Gamma^\dagger & 0 & H_S & \Delta \\ 0 & -\Gamma^\dagger & \Delta^\dagger & -H_S^* \end{pmatrix} \quad \begin{pmatrix} c \\ c^\dagger \\ b \\ b^\dagger \end{pmatrix}$$



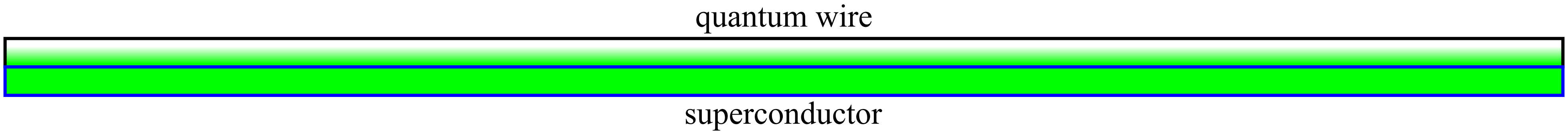
Effective descripton of the normal region ?

use second order quasi degenerate PT!

$$H^{Eff}_{BdG} = \begin{pmatrix} H_N^{'} & \Delta' \\ \Delta'^\dagger & -H_N^{'*} \end{pmatrix} \quad \begin{pmatrix} c \\ c^\dagger \end{pmatrix}$$

Proximity effect s-wave case

A simple model: 1D quantum wire on top of a 1D s-wave superconductor



$$H_{BdG} = \begin{pmatrix} \overbrace{\begin{bmatrix} \varepsilon_1 & \alpha & & \\ \alpha & \varepsilon_1 & & \\ & & -\varepsilon_1 & -\alpha \\ & & -\alpha & -\varepsilon_1 \end{bmatrix}}^{N} & \overbrace{\begin{bmatrix} t & & & \\ & t & & \\ & & -t & \\ & & & -t \end{bmatrix}}^{T} \\ \begin{bmatrix} t & & & \\ & t & & \\ & & -t & \\ & & & -t \end{bmatrix} & \underbrace{\begin{bmatrix} \varepsilon_2 & 0 & 0 & \Delta \\ 0 & \varepsilon_2 & -\Delta & 0 \\ 0 & -\Delta & -\varepsilon_2 & 0 \\ \Delta & 0 & 0 & -\varepsilon_2 \end{bmatrix}}_{S} \end{pmatrix} \begin{pmatrix} c_\uparrow \\ c_\downarrow \\ c_\uparrow^\dagger \\ c_\downarrow^\dagger \\ b_\uparrow \\ b_\downarrow \\ b_\uparrow^\dagger \\ b_\downarrow^\dagger \end{pmatrix}$$

$$\varepsilon_i = p^2/2m_i - \mu_i$$

Proximity effect s-wave case

Diagonalizing the superconductor:

$$H_{BdG} = \begin{pmatrix} N & T \\ T^\dagger & S \end{pmatrix} \rightarrow H'_{BdG} = \begin{pmatrix} N' & T' \\ T'^\dagger & S' \end{pmatrix}$$

$$S = \sqrt{\varepsilon_2^2 + \Delta^2} \begin{pmatrix} \cos(\varphi) & 0 & 0 & \sin(\varphi) \\ 0 & \cos(\varphi) & -\sin(\varphi) & 0 \\ 0 & -\sin(\varphi) & -\cos(\varphi) & 0 \\ \sin(\varphi) & 0 & 0 & -\cos(\varphi) \end{pmatrix},$$

$$\begin{pmatrix} \cos(\varphi) \\ \sin(\varphi) \end{pmatrix} = \frac{1}{\sqrt{\varepsilon_2^2 + \Delta^2}} \begin{pmatrix} \varepsilon_2 \\ \Delta \end{pmatrix}$$

$$S' = W^\dagger S W = \sqrt{\varepsilon_2^2 + \Delta^2} \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

$$W = \begin{pmatrix} \cos(\varphi/2) & 0 & 0 & -\sin(\varphi/2) \\ 0 & \cos(\varphi/2) & \sin(\varphi/2) & 0 \\ 0 & -\sin(\varphi/2) & \cos(\varphi/2) & 0 \\ \sin(\varphi/2) & 0 & 0 & \cos(\varphi/2) \end{pmatrix}$$

Transformed coupling:

$$T' = TW = t \begin{pmatrix} \cos(\varphi/2) & 0 & 0 & -\sin(\varphi/2) \\ 0 & \cos(\varphi/2) & \sin(\varphi/2) & 0 \\ 0 & \sin(\varphi/2) & -\cos(\varphi/2) & 0 \\ -\sin(\varphi/2) & 0 & 0 & -\cos(\varphi/2) \end{pmatrix}$$

Proximity effect s-wave case

Apply second order quasi degenerate perturbation theory:

$$H^{(0)}_{mm'} = H^{0}_{mm'} \qquad H^{(2)}_{mm'} = \frac{1}{2}\sum_{l} \frac{H'_{ml}}{E_m - E_l} H'_{lm'} + H'_{ml} \frac{H'_{lm'}}{E_{m'} - E_l}$$
$$H^{(1)}_{mm'} = H^{'}_{mm'}$$

Effective BdG operator for normal part:

$$\Delta_{eff} \approx \frac{t^2}{\Delta}$$

s-wave characteristics is inherited!

$$N' \approx \underbrace{\begin{pmatrix} \varepsilon_1 & \alpha & & \\ \alpha & \varepsilon_1 & & \\ & & -\varepsilon_1 & -\alpha \\ & & -\alpha & -\varepsilon_1 \end{pmatrix}}_{H^{(0)}+H^{(1)}}$$
$$+ \underbrace{\frac{1}{\varepsilon_2^2 + \Delta^2 - \varepsilon_1^2} \begin{pmatrix} -(\varepsilon_1+\varepsilon_2)t^2 & & & t^2\Delta \\ & -(\varepsilon_1+\varepsilon_2)t^2 & -t^2\Delta & \\ & -t^2\Delta & (\varepsilon_1+\varepsilon_2)t^2 & \\ t^2\Delta & & & (\varepsilon_1+\varepsilon_2)t^2 \end{pmatrix}}_{H^{(2)}}$$

Andreev reflection in s- and p-wave

s-wave

$$H^s_{BdG} = \begin{pmatrix} -2t\cos(k)-\mu & & & \Delta \\ & -2t\cos(k)-\mu & -\Delta & \\ & -\Delta & 2t\cos(k)+\mu & \\ \Delta & & & 2t\cos(k)+\mu \end{pmatrix}$$

$$H^s_{BdG}|_{e\to h} = \begin{pmatrix} p & & & \Delta \\ & p & -\Delta & \\ & -\Delta & -p & \\ \Delta & & & -p \end{pmatrix}$$

$$H^s_{BdG}|_{h\to e} = \begin{pmatrix} -p & & & \Delta \\ & -p & -\Delta & \\ & -\Delta & p & \\ \Delta & & & p \end{pmatrix}$$

p-wave

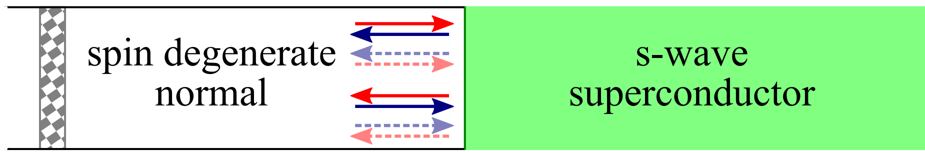
$$H^p_{BdG} = \begin{pmatrix} -2t\cos(k)-\mu & \Delta\sin(k) \\ \Delta\sin(k) & 2t\cos(k)+\mu \end{pmatrix}$$

$$H^p_{BdG}|_{e\to h} = \begin{pmatrix} p & \tilde{\Delta} \\ \tilde{\Delta} & -p \end{pmatrix}$$

$$H^p_{BdG}|_{h\to e} = \begin{pmatrix} -p & -\tilde{\Delta} \\ -\tilde{\Delta} & p \end{pmatrix}$$

non-universal

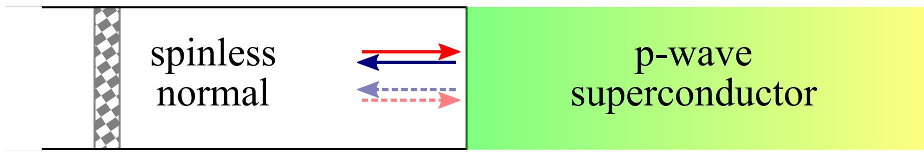
$$\left.\begin{matrix} r_{e\to h}(+\Delta),\ r_{h\to e}(+\Delta) \\ + \\ r_{e\to h}(-\Delta),\ r_{h\to e}(-\Delta) \end{matrix}\right\} \Rightarrow G(0) = 0\ldots 4\frac{e^2}{h}$$



universal!!!

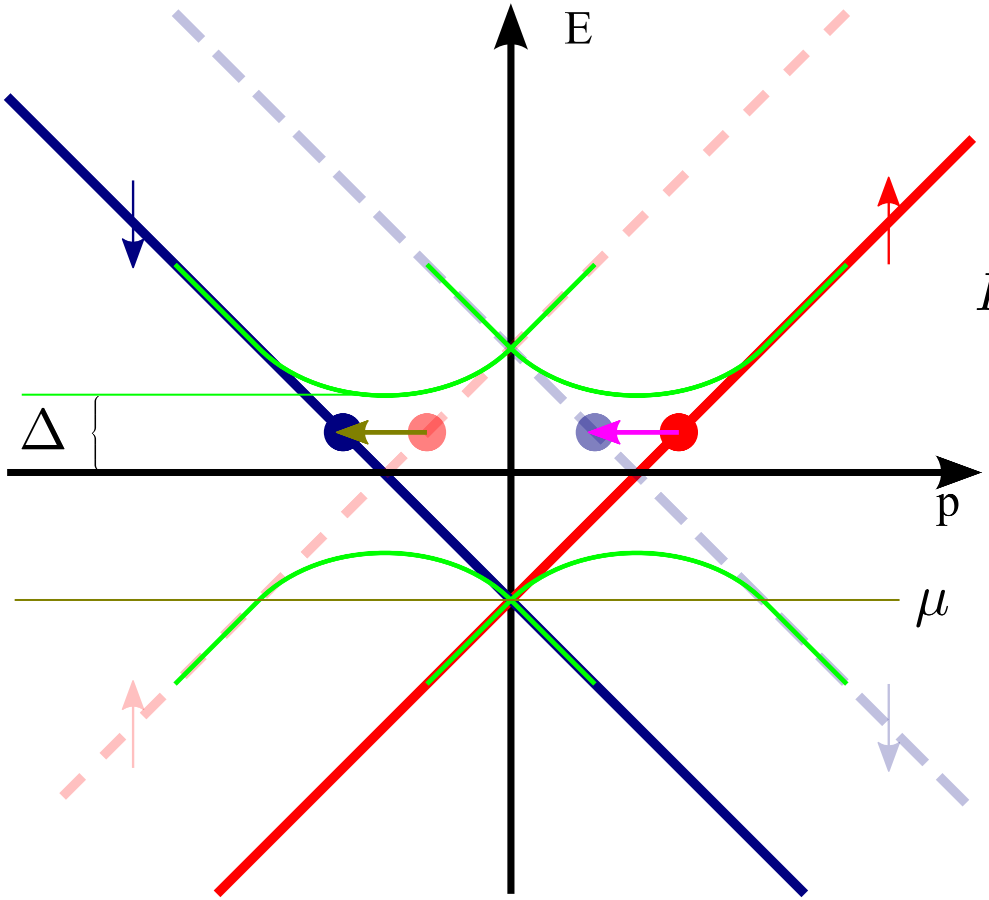
$$\left.\begin{matrix} r_{e\to h}(+\Delta) \\ + \\ r_{h\to e}(-\Delta) \end{matrix}\right\} \Rightarrow G(0) = 2\frac{e^2}{h}$$

@ finite voltage (energy): universality does not hold due to the same additional phase gained by e-s and h-s



Effective p-wave pairing from s-wave

HgTe edge in proximity to an s-wave superconductor

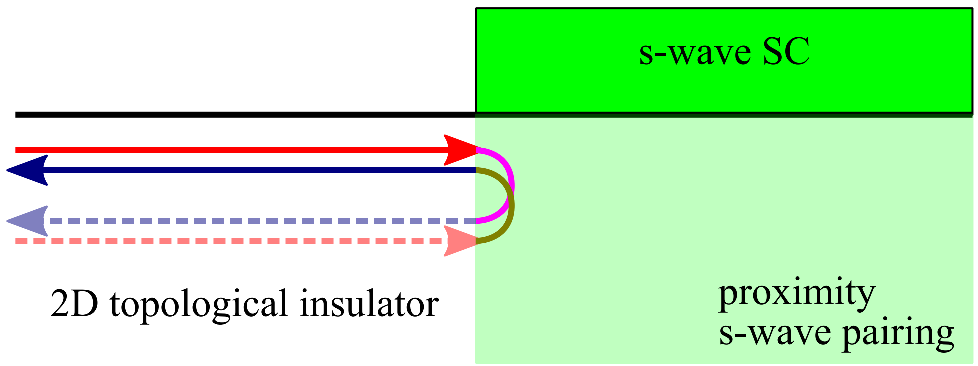


$$H_{BdG} = \begin{pmatrix} H & \Delta \\ -\Delta^* & -H^* \end{pmatrix}, \qquad H = p\sigma_z - \mu\sigma_0$$

$r_{e\to h}$

$$H_{BdG} = \begin{pmatrix} p-\mu & 0 & 0 & \Delta \\ 0 & -p-\mu & -\Delta & 0 \\ 0 & -\Delta & p+\mu & 0 \\ \Delta & 0 & 0 & -p+\mu \end{pmatrix}$$

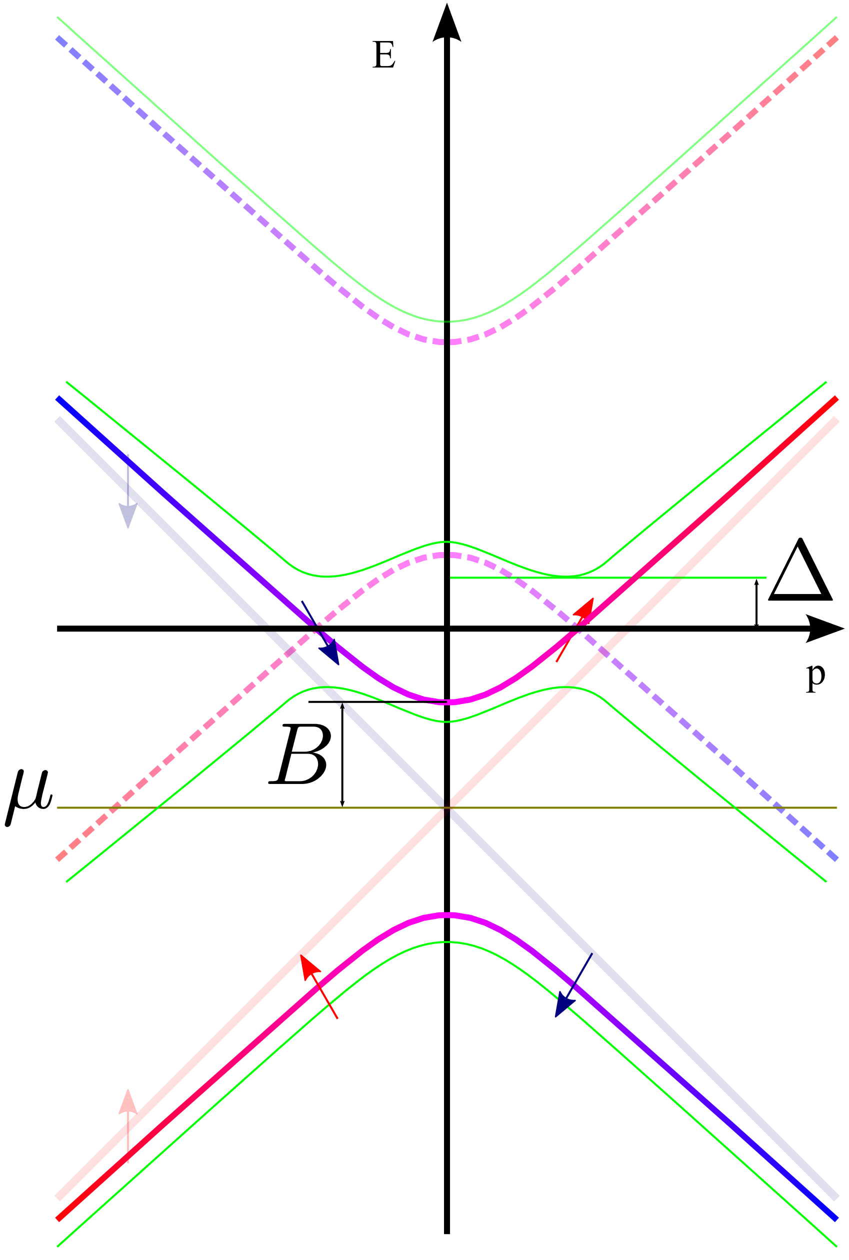
$r_{h\to e}$



Because of the chiral nature of electrons Andreev and inverse Andreev reflection have different phase!
ZBCP expected in NIS junction!!

L. Fu and C. L. Kane, Phys. Rev. B, **79**, 161408(R) (2009).

Effective p-wave pairing from s-wave



Effect of a Finite Zeeman splitting, phase transition

$$H = p\sigma_z + B\sigma_x - \mu\sigma_0$$

$$H_{BdG} = \begin{pmatrix} p-\mu & B & 0 & \Delta \\ B & -p-\mu & -\Delta & 0 \\ 0 & -\Delta & p+\mu & -B \\ \Delta & 0 & -B & -p+\mu \end{pmatrix}$$

diagonalizing the normal region

$$\tilde{H}_{BdG} = \begin{pmatrix} \sqrt{\,}-\mu & & -\Delta B/\sqrt{\,} & \Delta p/\sqrt{\,} \\ & -\sqrt{\,}-\mu & -\Delta p/\sqrt{\,} & -\Delta B/\sqrt{\,} \\ -\Delta B/\sqrt{\,} & -\Delta p/\sqrt{\,} & \sqrt{\,}+\mu & \\ \Delta p/\sqrt{\,} & -\Delta B/\sqrt{\,} & & -\sqrt{\,}+\mu \end{pmatrix}$$

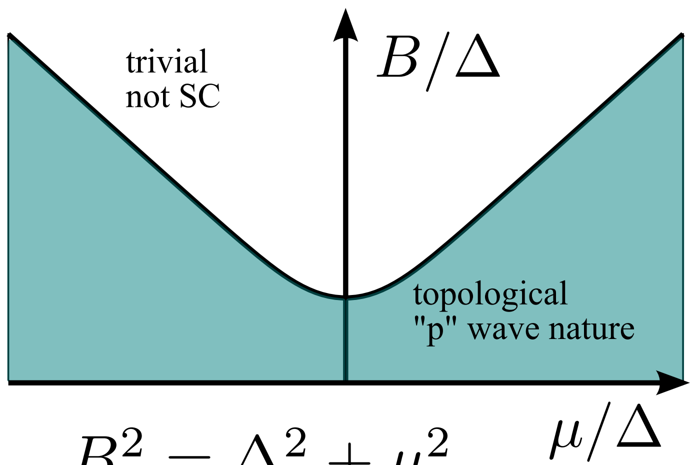
$$\sqrt{\,} = \sqrt{p^2+B^2}$$

$$\underbrace{r_{e\to h}(+\Delta), r_{h\to e}(-\Delta)}$$

$$\Downarrow$$

$$G(0) = 2\frac{e^2}{h}$$

universal!!!



$$B^2 = \Delta^2 + \mu^2$$

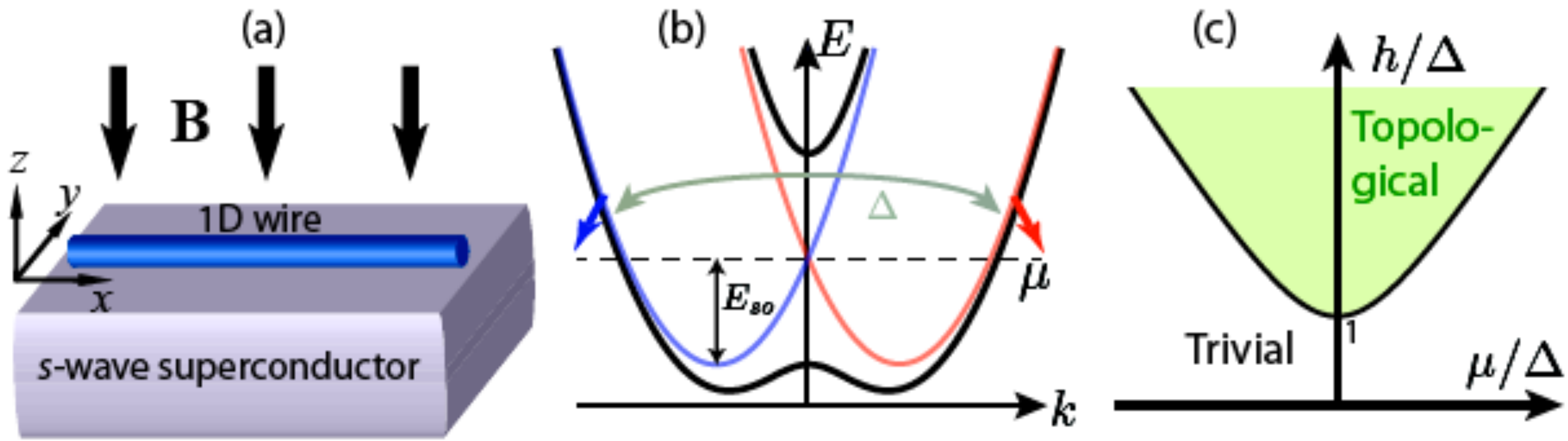
L. Fu and C. L. Kane, Phys. Rev. B, **79**, 161408(R) (2009).

Lutchyn's wire

1D quantum wire with spin orbit coupling in a magnetic field on top of a superconductor

$$H_{wire} = p^2 - \mu + h\sigma_z + p\alpha\sigma_y$$

$$E_{\pm} = p^2 \pm \sqrt{h^2 + \alpha^2 p^2} - \mu$$



R. M. Lutchyn, J. D. Sau, and S. Das Sarma, Phys. Rev. Lett., **105**, 077001 (2010).
Y. Oreg, G. Refael, and F. von Oppen, Phys. Rev. Lett., **105**, 177002 (2010).

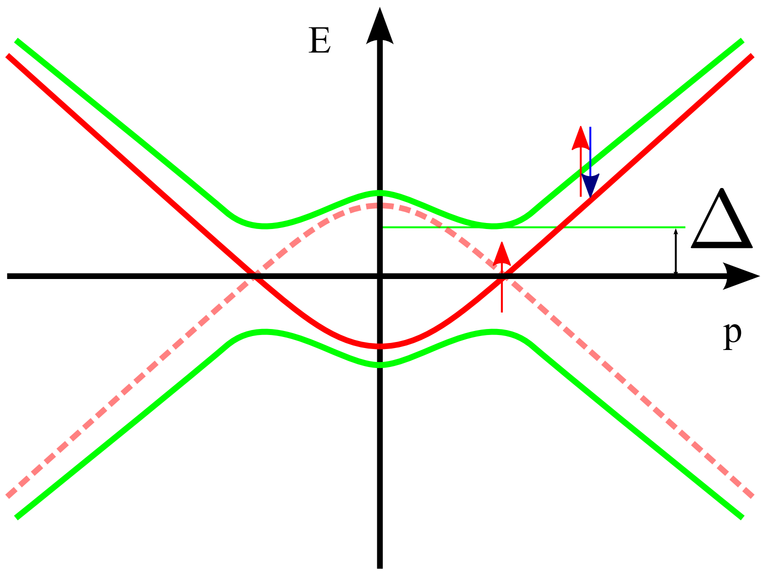
Four possible junctions and smoking guns

# of spins\type of SC	trivial	topological
spin polarized	$G(0) = 0$	$G(0) = 2e^2/h$
two spin	$G(0) = 0 \ldots 4e^2/h$	$G(0) = 2e^2/h$

Invoke adiabatic equivalence!

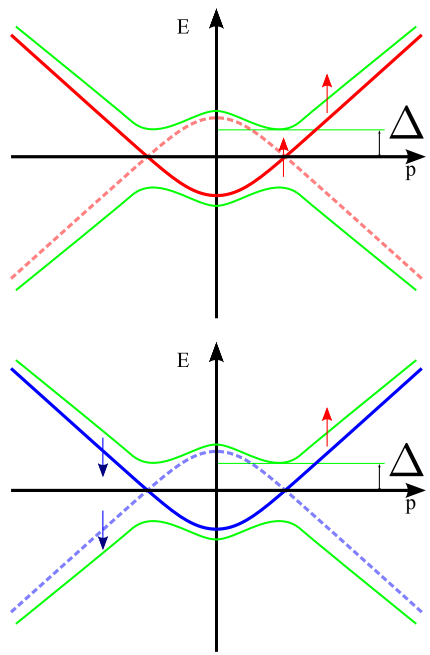
Andreev reflection needs spin flip!

Everything will reflect normally!



2 spins work differently

One perfect Andreev!



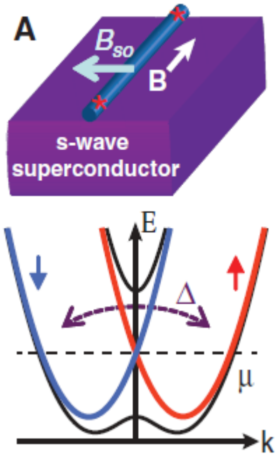
Same as Kitaev

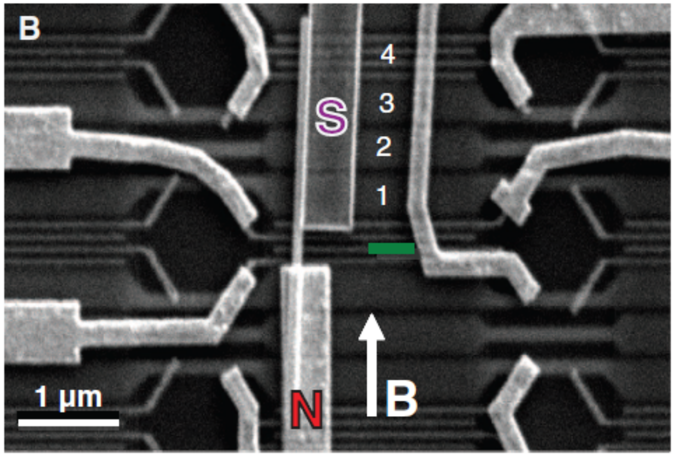
Everything will Andreev reflect!

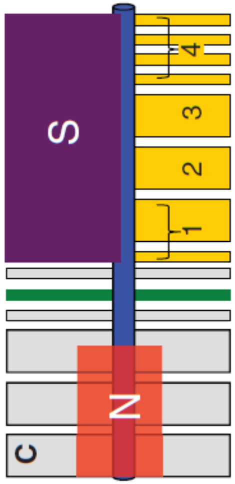
Superconductor does not talk to this spin!

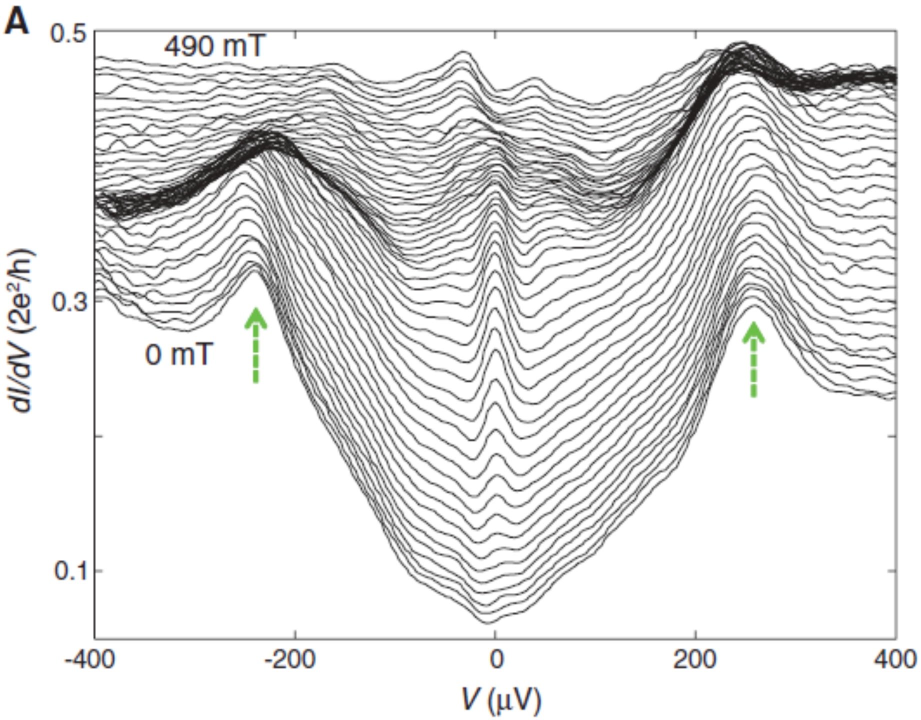
Everything will reflect normally!

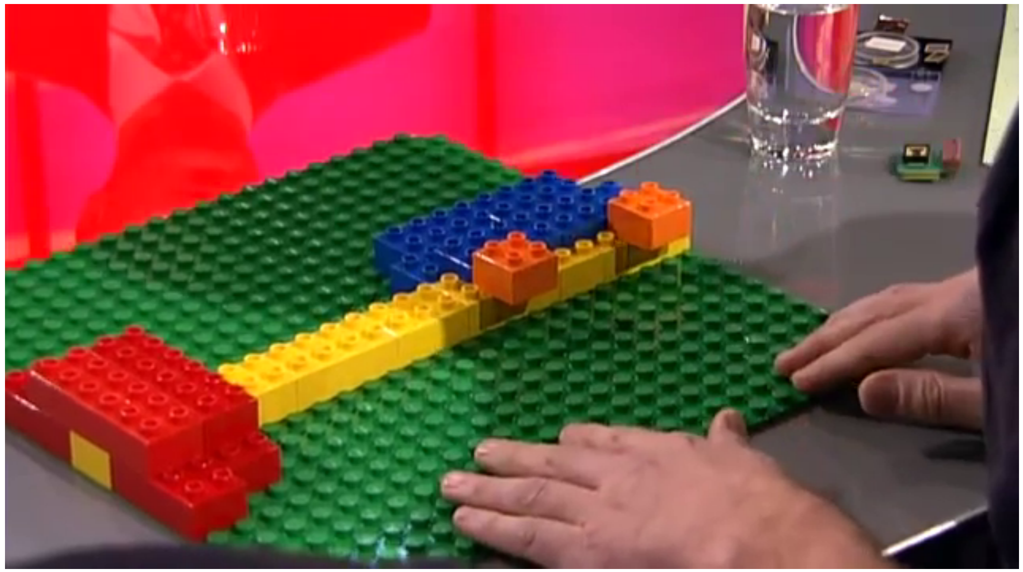
Delft experiment











V. Mourik, K. Zuo, S. M. Frolov, S. R. Plissard, E. P. A. M. Bakkers, L. P. Kouwenhoven, Science, **336**, 1003 (2012)

Delft experiment