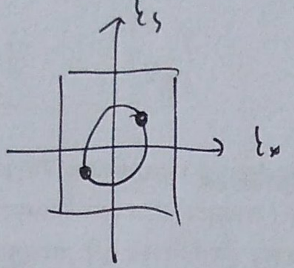
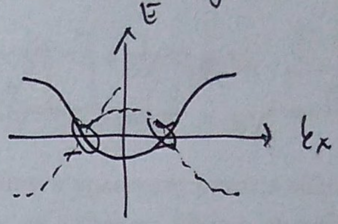
Andreev reflection

A 1

1) Formation of a Cooper pair = incoming electron → reflected hole



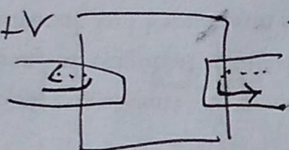


~~2)~~ 1) Hole ~~ds~~ left by taking e^- @ $+k$, ↑, right-mover: $-k$, ↓, right-mover

2) Andreev reflection keeps k constant!

3) Inverse Andreev: hole → electron. Takes place e.g. during conductance

Fermi sea ↑ extra electrons

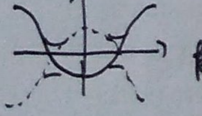


Fermi sea ↓ extra holes

3) Andreev reflection is coherent. Amplitude obtained by BTK

$$\boxed{\begin{aligned} k_F:&\quad r_A = e^{-i\varphi}\, e^{-i\arccos\frac{E}{\Delta}} \\ -k_F:&\quad \tilde{r}_A = e^{+i\varphi}\, e^{-i\arccos E/\Delta} \end{aligned}}$$

s-wave: $$\hat{H} = \left(\hat{c}^{\dagger}_{k\uparrow}\ \hat{c}_{-k\downarrow}\right)\begin{bmatrix} -2t\cos k-\mu & \Delta(x) \\ \Delta^*(x) & 2t\cos k+\mu \end{bmatrix}\begin{pmatrix}\hat{c}_{k\uparrow} \\ \hat{c}^{\dagger}_{-k\downarrow}\end{pmatrix}$$



(above gap: HW) — Linearize — EFA $$H = \begin{bmatrix} -iv_F\partial_x & \Delta \\ \Delta^* & iv_F\partial_x \end{bmatrix}$$

Inside gap: ~~linearize~~ $e^{ik_F x}\, e^{-Kx}$

$$H = \begin{bmatrix} ivK & \Delta \\ \Delta^* & -ivK \end{bmatrix} = ivK\sigma^z + \mathrm{Re}\,\Delta\,\sigma^x - \mathrm{Im}\,\Delta\,\sigma^y \qquad \Delta = |\Delta| e^{i\varphi}$$

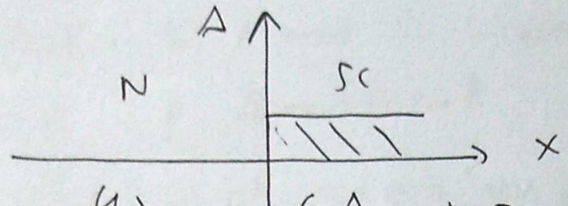
$$H^2 = \sqrt{|\Delta|^2 - v^2K^2} \qquad vK = \pm\sqrt{|\Delta|^2 - E^2}$$

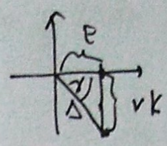
penetration depth diverges @ $\Delta = E$

need only + energy: $$\begin{pmatrix}\Delta \\ E - ivK\end{pmatrix} = \begin{pmatrix}u \\ v\end{pmatrix}$$

$|u|^2 = |v|^2$ electron-hole weight equal

A2





$+k$, $+v$ $\quad \begin{pmatrix}1\\ r_A\end{pmatrix} \quad \begin{pmatrix}\Delta\\ E-ivK\end{pmatrix} \Big\} \Rightarrow r_A = \frac{E-ivK}{\Delta} = e^{-i\varphi} e^{-i\arccos\frac{E}{|\Delta|}}$

Inverse Andreev:

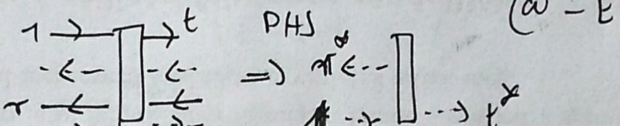
$-k$, $-v$ $\quad \begin{pmatrix}\tilde{r}_A\\ 1\end{pmatrix} \quad \begin{pmatrix}\Delta\\ E+ivK\end{pmatrix} \Big\} \Rightarrow \tilde{r}_A = \left(\frac{\Delta}{E+ivK}\right) = e^{+i\varphi} e^{-i\arccos\frac{E}{|\Delta|}}$

other valley

4) Conductance across a tunnel junction weaker in SC

This can be used to measure gap.

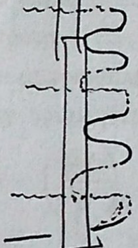
Scattering matrix:



Calculate Andreev:

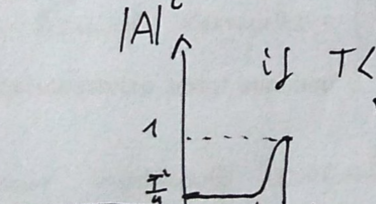
Naïve: $r_A \sim |t|^2$

Correction due to multiple scattering:



$$A = t'^* r_A \left[1 - r' \tilde{r}_A r'^* r_A\right] t$$

single mode: $|A| \approx \left| r_A \frac{t'^* t}{1 - e^{-2i\arccos\frac{E}{\Delta}} |r'|^2} \right| = \frac{T}{1 - e^{-2i\arccos\frac{E}{\Delta}} R}$



if $T \ll 1$: $\cancel{1 - e^{-2i\arccos\frac{E}{\Delta}} R} \gg T$ ex

$E = 0$: $|A| = \frac{T}{1+R} = \frac{T}{2-T}$

$E = |\Delta|$: $|A| = \frac{T}{1-R} = 1$

$\rightarrow 1 - e^{-2i\arccos\frac{E}{\Delta}} R \gg T$ except $\frac{vK}{\Delta} < T$

~~Ran~~

$$\left.\begin{array}{l} \underbrace{v^2K^2}_{=\,\Delta^2 - E^2 \,=\, (\Delta+E)(\Delta-E) \,\simeq\, 2\Delta(\Delta-E)} < T^2\Delta^2 \end{array}\right\} \Delta - E < \frac{T^2}{2} \cdot \Delta$$

5) Application to p-wave SC : use ~~BdG~~ via BdG $\boxed{A\,3}$

$\Rightarrow \quad \Delta \to i\Delta \sin k$

$$\hat{H} = \sum_k \left(t\, 2\cos k\, c_k^+ c_k - \mu\, c_k^+ c_k \right) + \Delta \sum_k e^{ik} c_k^+ c_{-k}^+ + \Delta^* \sum_k e^{-ik} c_{-k} c_k$$

only $\sin k$ $\qquad$ only $\sin k$ remains

$\Downarrow$

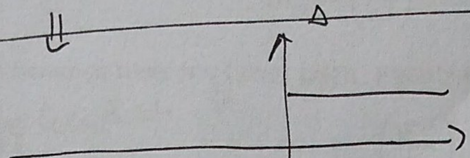
$$\mathcal{H}(k) = \begin{bmatrix} -t\cos k - \mu/2 & i\Delta \sin k \\ -i\Delta^* \sin k & t\cos k + \mu/2 \end{bmatrix}$$

$\cancel{E = \sqrt{|\Delta|^2(1+v_\Delta k)^2 - v^2k^2}}$

$\Downarrow$

6)

$\Delta(k_F) = i\Delta \sin k_F$

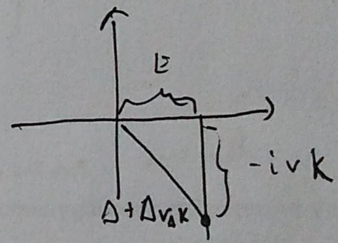


$$\begin{pmatrix} 1 \\ r_A \end{pmatrix} \qquad \begin{pmatrix} \Delta^{(k_F)} \\ E - ivK \end{pmatrix} \qquad r_A = e^{-i\varphi}\, e^{-i\cdots}$$

$$\begin{pmatrix} \tilde{r}_A \\ 1 \end{pmatrix} \qquad \begin{pmatrix} -\Delta^{(k_F)} \\ E + ivK \end{pmatrix} \qquad \tilde{r}_A = -e^{i\varphi}\, e^{-i\cdots}$$

$\Downarrow$

$$|A| = \frac{T}{1 + e^{2i\cdots} \cdot R}$$



@ $E = 0$: $\quad \cancel{\left(\sqrt{v^2 - |\Delta|^2 v_\Delta^2}\right) K}$

Narrow resonance: $\dfrac{E^2}{v^2K^2} < E_T^2$

Narrow resonance around $E = 0$: $|A| = 1$ $\qquad \boxed{\text{ZBCP}}$

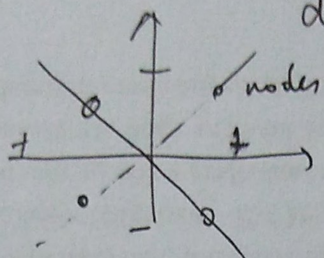
@ $E = \Delta$: $\quad K = 0$

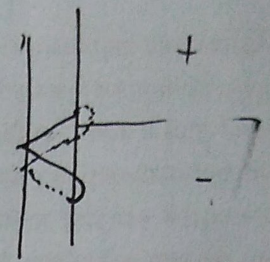
$$|A| = \frac{T}{1 + R}$$

7)

7) Application to 2D d-wave (cuprates): ZBCP

$d_{x^2-y^2}$: $\Delta \to \Delta(\cos k_x - \cos k_y)$

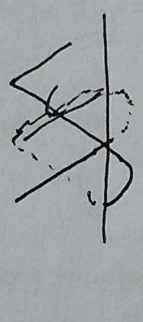




depending on angle of surface

8) Application to 2D p+ip (Sr_2CuO_4 ?)

$\Delta \to \Delta \sin k_x + i \sin k_y$



$\varphi \to \varphi \cdot k_{x1} \to$ displacement during Andreev reflection

same direction displacement

@ every Andreev

$\Downarrow$

edge state!

Chern # $\Rightarrow$ Propagating Majorana modes