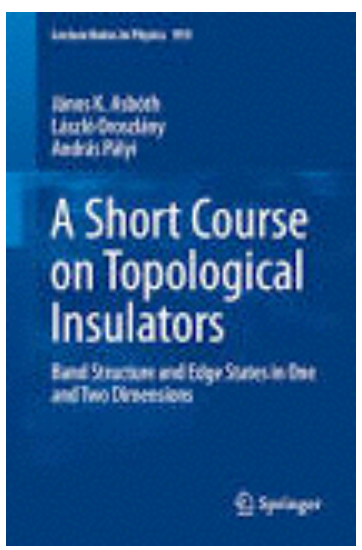
A simple model for a topological quantum memory

Andras Palyi
Budapest University of Technology and Economics

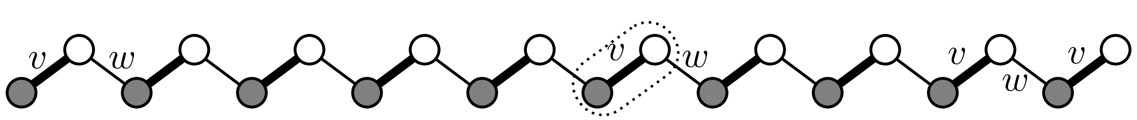
From Topology to Superconductivity, Vienna, 2018.02.23

Part 1
Introduction to topological insulators

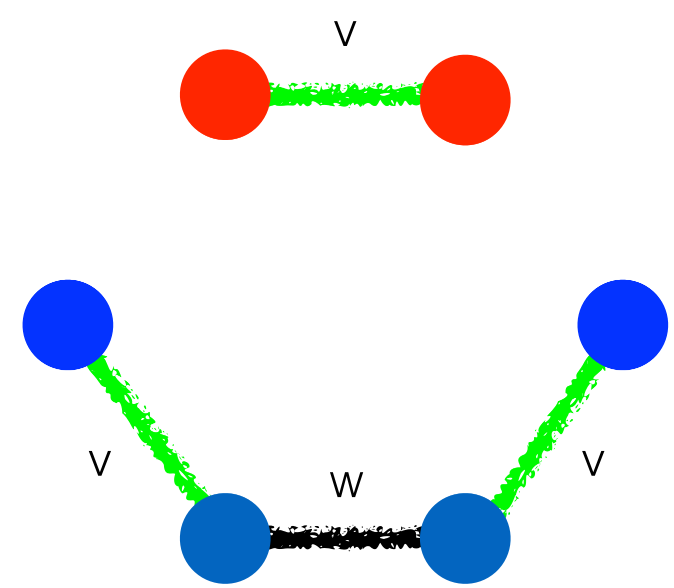


arXiv:1509.02295

(here: Chapter 1)



Part 2
A topological quantum memory



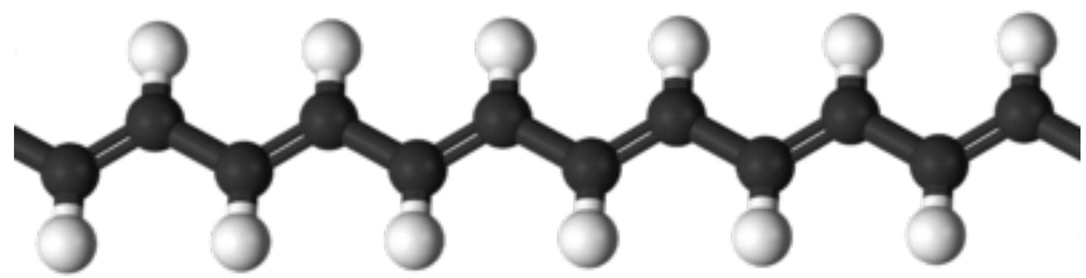
with:
Janos Asboth, Peter Boross, Laszlo Oroszlany, Gabor Szechenyi

Part 1
Introduction to topological insulators

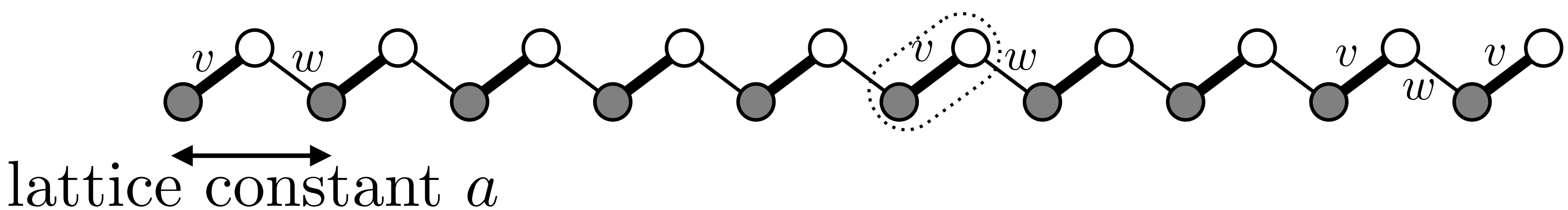
If the bulk has nontrivial topology,
then the edge has disorder-resistant bound states
(‘bulk-boundary correspondence’)

SSH is a tight-binding toy model for polyacetylene

Polyacetylene



Su-Schrieffer-Heeger (SSH) model of polyacetylene



Real-space tight-binding SSH Hamiltonian:

$$\hat{H} = v \sum_{m=1}^{N} \left(|m,B\rangle \langle m,A| + h.c. \right) + w \sum_{m=1}^{N-1} \left(|m+1,A\rangle \langle m,B| + h.c. \right).$$

intracell hopping

intercell hopping

For N=4:

$$H = \begin{pmatrix} 0 & v & 0 & 0 & 0 & 0 & 0 & 0 \\ v & 0 & w & 0 & 0 & 0 & 0 & 0 \\ 0 & w & 0 & v & 0 & 0 & 0 & 0 \\ 0 & 0 & v & 0 & w & 0 & 0 & 0 \\ 0 & 0 & 0 & w & 0 & v & 0 & 0 \\ 0 & 0 & 0 & 0 & v & 0 & w & 0 \\ 0 & 0 & 0 & 0 & 0 & w & 0 & v \\ 0 & 0 & 0 & 0 & 0 & 0 & v & 0 \end{pmatrix}$$

k-space Hamiltonian maps unit circle to complex plane

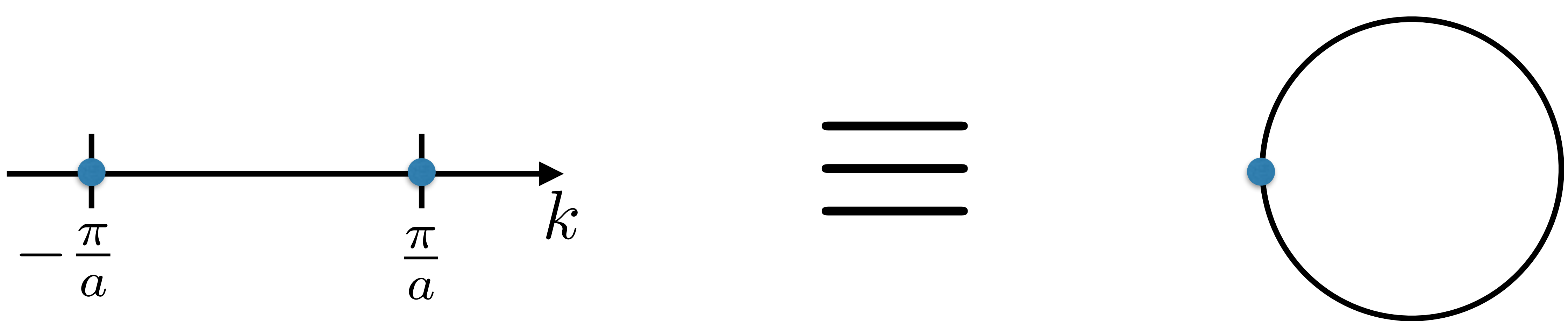
Real-space tight-binding SSH Hamiltonian:

$$\hat{H} = v \sum_{m=1}^{N} \left(|m,B\rangle \langle m,A| + h.c. \right) + w \sum_{m=1}^{N-1} \left(|m+1,A\rangle \langle m,B| + h.c. \right).$$

k-space SSH Hamiltonian:

$$H(k) = \begin{pmatrix} 0 & v + we^{-ik} \\ v + we^{ik} & 0 \end{pmatrix}$$

Brillouin zone of a 1D crystal is equivalent to the unit circle:



$$f_{v,w} : \text{unit circle} \to \mathbb{C},\, k \mapsto v + we^{-ik}$$

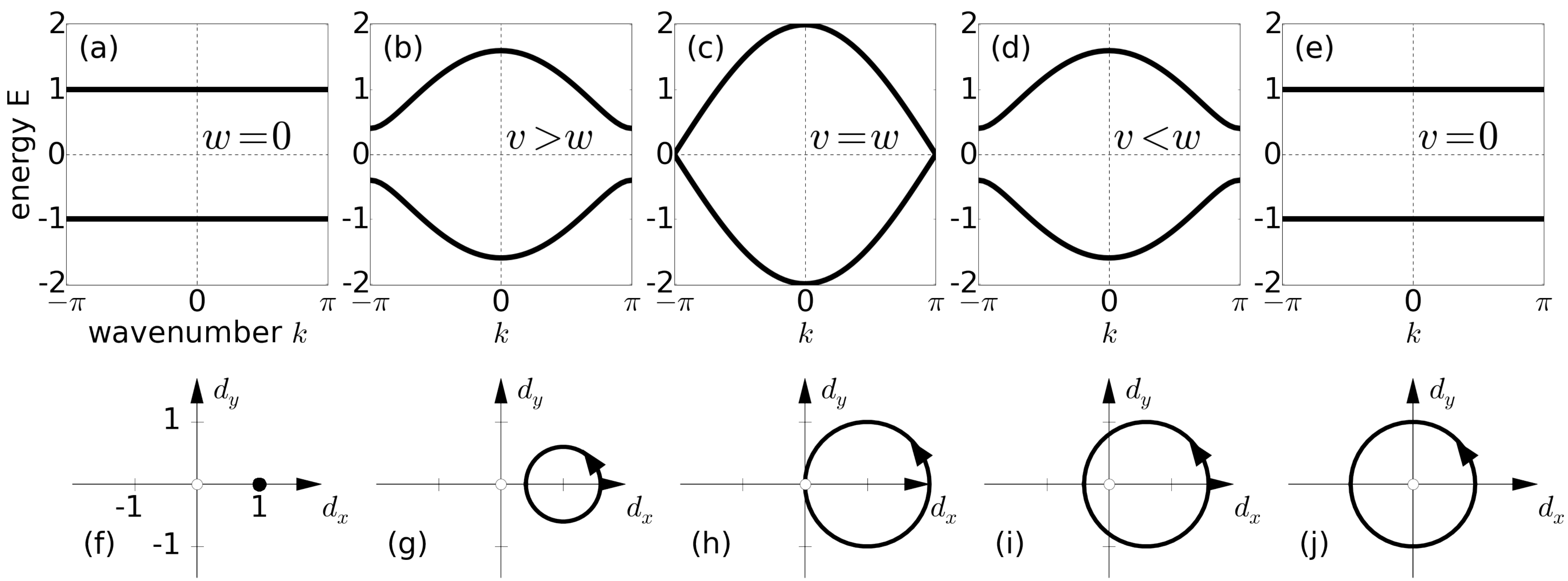
An insulating SSH Hamiltonian has a topological invariant

k-space SSH Hamiltonian:

$$H(k) = \begin{pmatrix} 0 & v + we^{-ik} \\ v + we^{ik} & 0 \end{pmatrix}$$

band structure, valence (-) and conduction (+) bands:

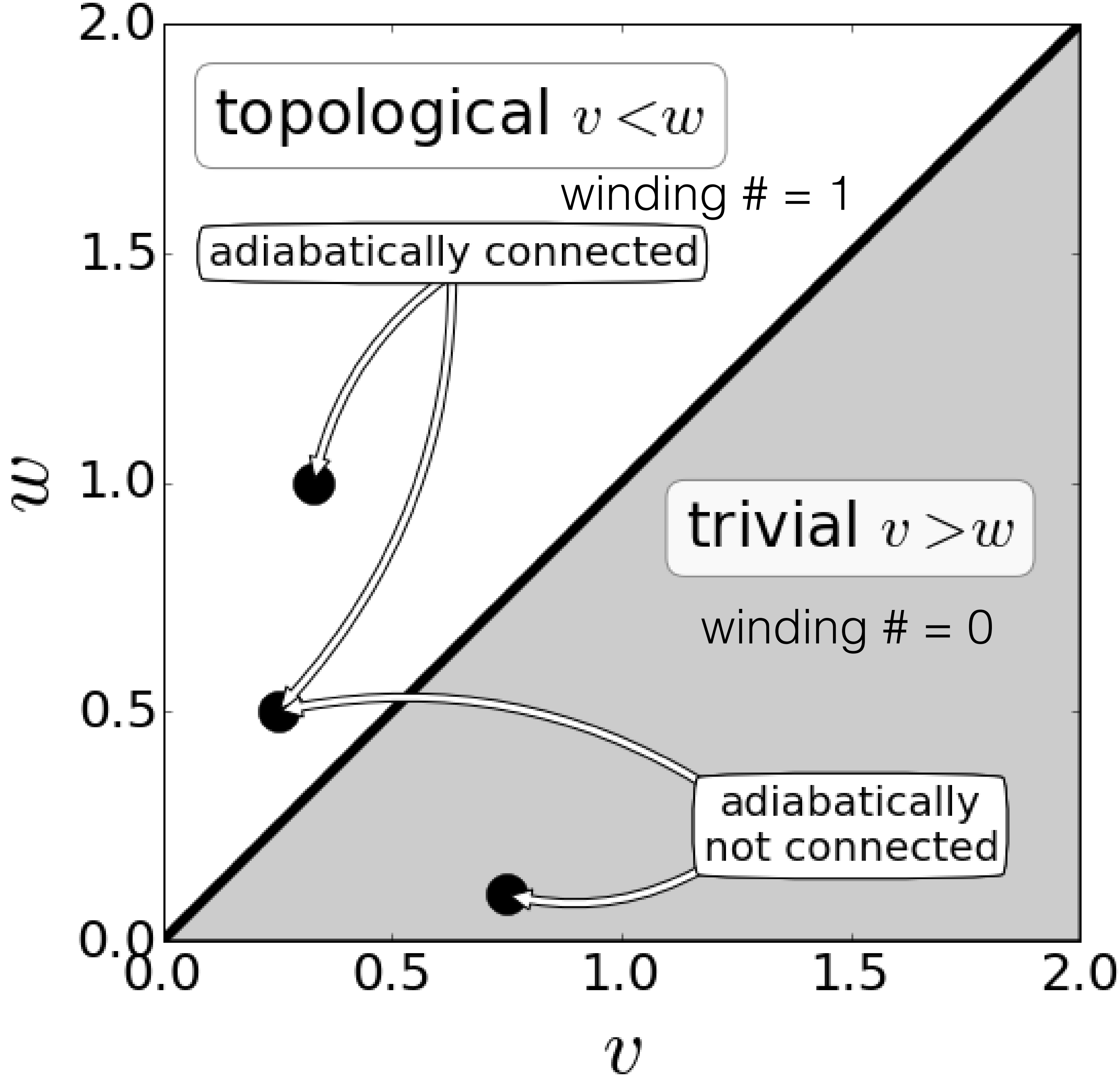
$$E(k) = \pm|f_{v,w}(k)| = \pm|v + we^{-ik}| = \pm\sqrt{v^2 + w^2 + 2vw\cos(k)}$$



insulator	insulator	metal	insulator	insulator

winding number → 0 0 1 1

SSH parameter space has two topological phases



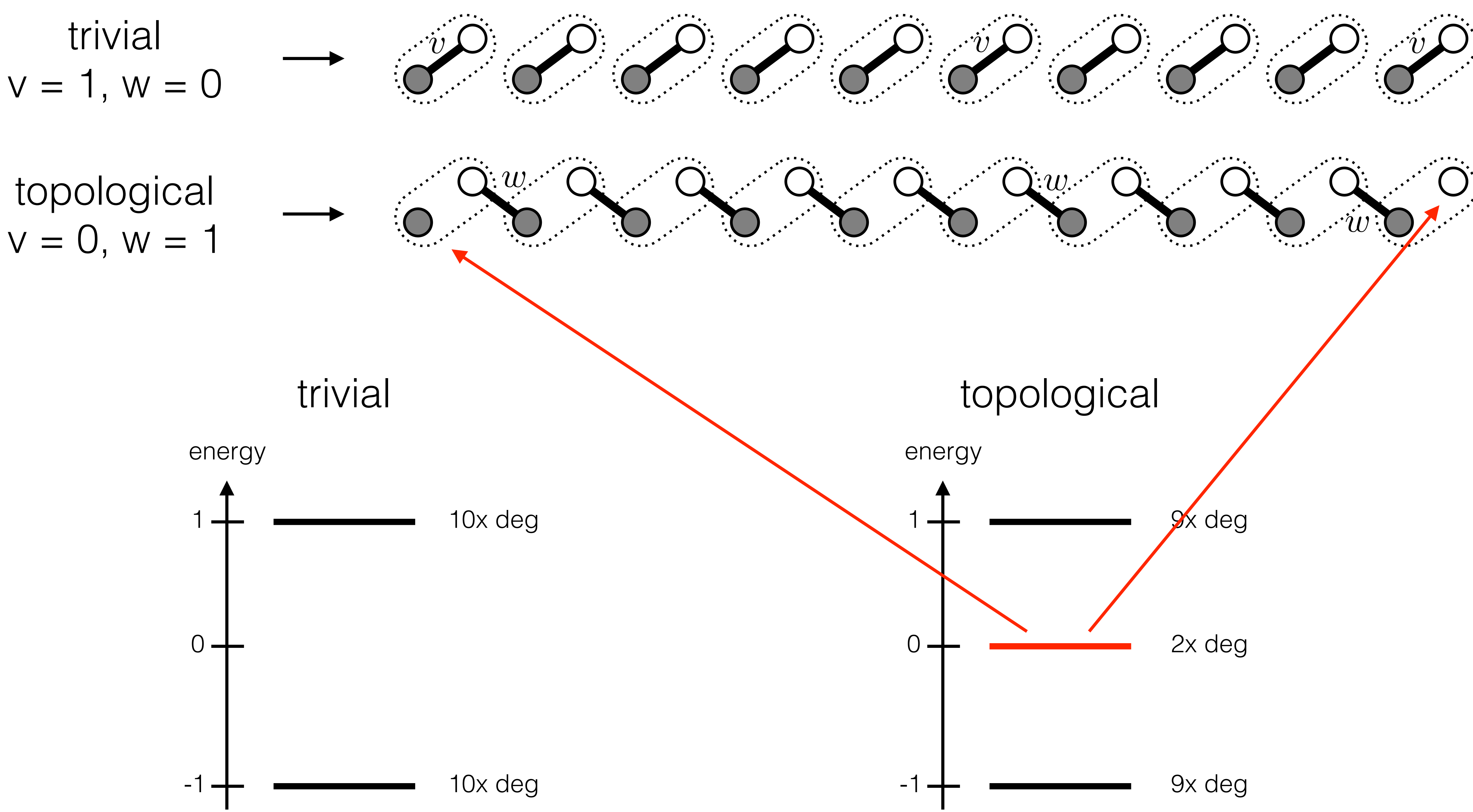
Part 1

Introduction to topological insulators

If the bulk has nontrivial topology,
then the edge has disorder-resistant bound states
(‘bulk-boundary correspondence’)

Zero intracell hopping implies zero-energy states at edges

Fully dimerized limits of the SSH Hamiltonian:



SSH Hamiltonians have chiral symmetry

Definition: a Γ local unitary operator is a *chiral symmetry* if $\Gamma H \Gamma^\dagger = -H$

SSH Hamiltonians have chiral symmetry:

For example, N = 4:

$$H = \begin{pmatrix} 0 & v & 0 & 0 & 0 & 0 & 0 & 0 \\ v & 0 & w & 0 & 0 & 0 & 0 & 0 \\ 0 & w & 0 & v & 0 & 0 & 0 & 0 \\ 0 & 0 & v & 0 & w & 0 & 0 & 0 \\ 0 & 0 & 0 & w & 0 & v & 0 & 0 \\ 0 & 0 & 0 & 0 & v & 0 & w & 0 \\ 0 & 0 & 0 & 0 & 0 & w & 0 & v \\ 0 & 0 & 0 & 0 & 0 & 0 & v & 0 \end{pmatrix} \qquad \Gamma = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

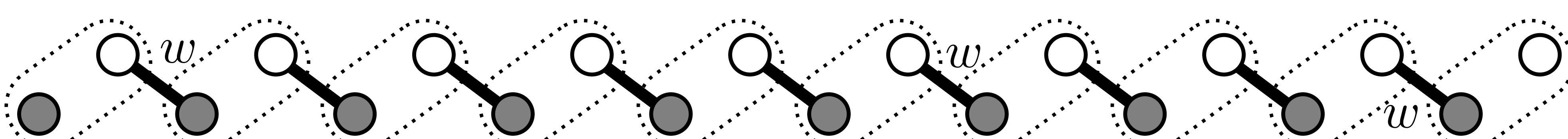
Consequences of chiral symmetry

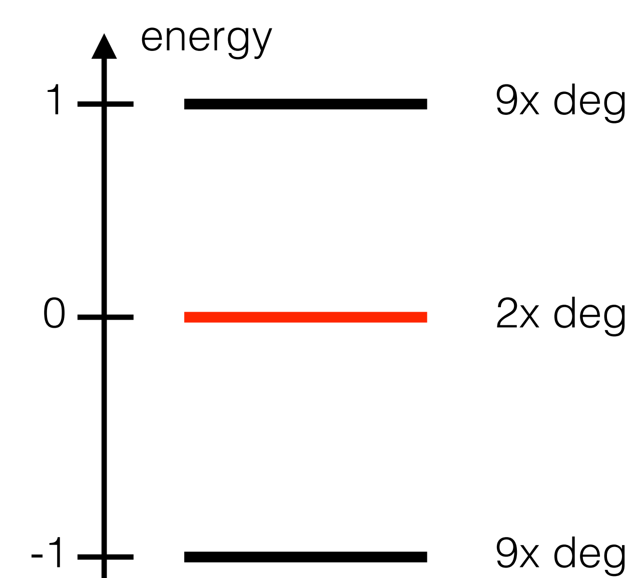
Up-down symmetric energy spectrum: $H\psi = E\psi$ implies $H(\Gamma\psi) = -E(\Gamma\psi)$

Finite-energy eigenstates have a 'chiral partner' at opposite energy

A zero-energy eigenstate might be its own chiral partner

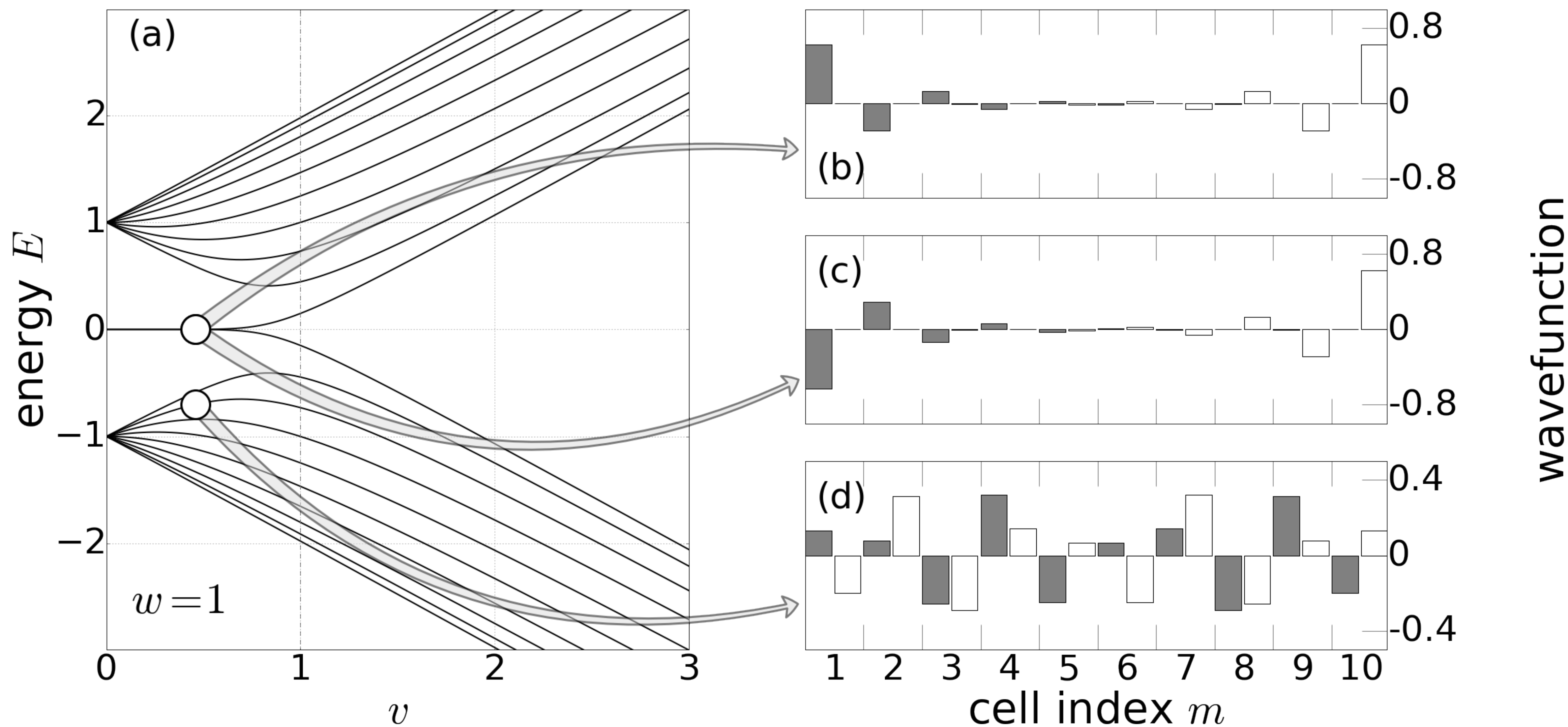
Example: fully dimerized topological SSH chain





Chiral symmetry implies edge states in topological SSH

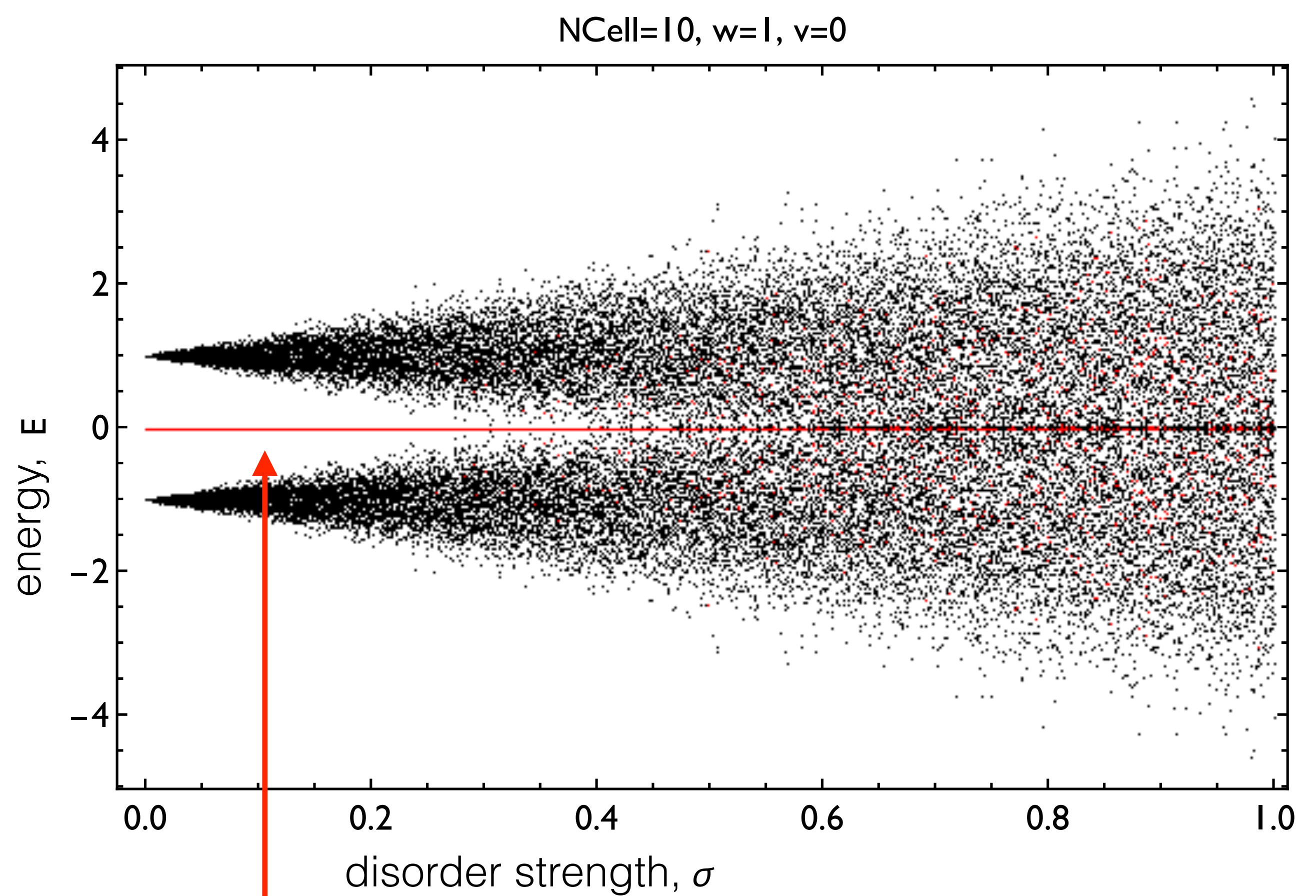
Take long fully dimerized topological SSH chain (v=0, w=1).
Switch on a uniform intercell hopping v.
Does the zero-energy edge state survive?
It does: its energy sticks to zero due to chiral symmetry.
The energy can leave zero only if the left and right edge states hybridize.



bulk-boundary correspondence

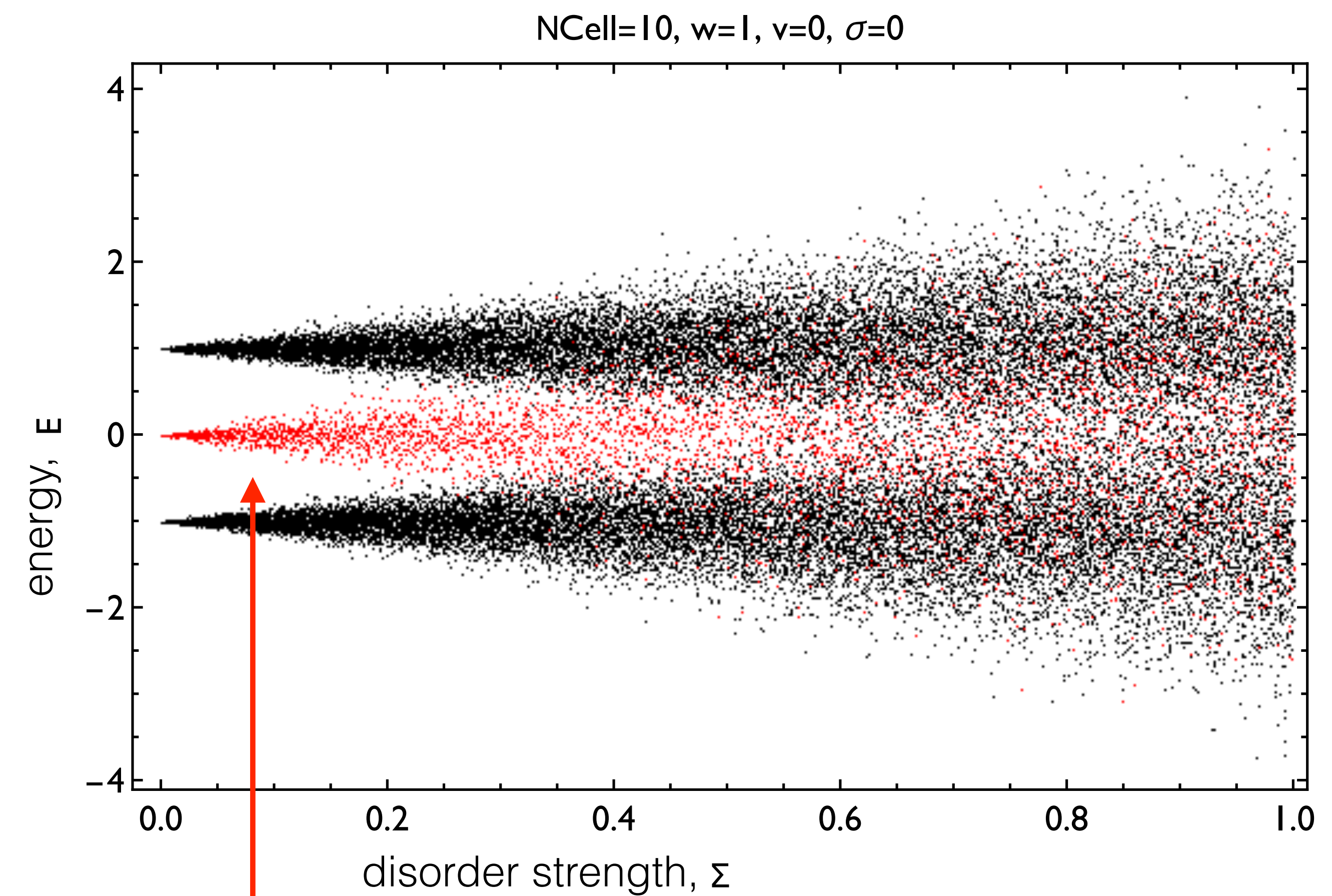
Edge states are robust against chiral-symmetric disorder

Hopping disorder
(respects chiral symmetry)



zero-energy edge states
survive disorder

On-site disorder
(breaks chiral symmetry)



zero-energy edge states
dissolved by disorder

SSH is one creature in the zoo of topological insulators

spatial dimensions

symmetry classes

Cartan\\d	1	2	3	
Complex case:				
A	0	$\mathbb{Z}$	0	quantum Hall effect
AIII	$\mathbb{Z}$	0	$\mathbb{Z}$	
Real case:				
AI	0	0	0	
BDI	$\mathbb{Z}$	0	0	SSH model
D	$\mathbb{Z}_2$	$\mathbb{Z}$	0	
DIII	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	
AII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	quantum spin Hall effect
CII	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	
C	0	$2\mathbb{Z}$	0	
CI	0	0	$2\mathbb{Z}$	

Table from A. W. W. Ludwig, Physica Scripta (2016)

Part 1
Introduction to topological insulators

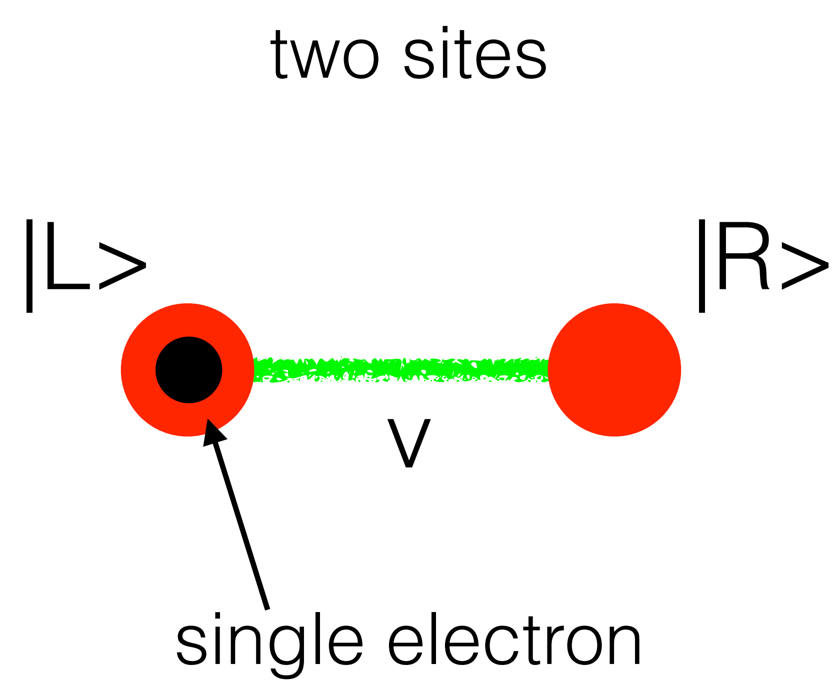
**If the bulk has nontrivial topology,
then the edge has disorder-resistant bound states
(‘bulk-boundary correspondence’)**

Part 2

A topological quantum memory

An almost noiseless qubit is obtained by putting together many noisy qubits

The charge qubit



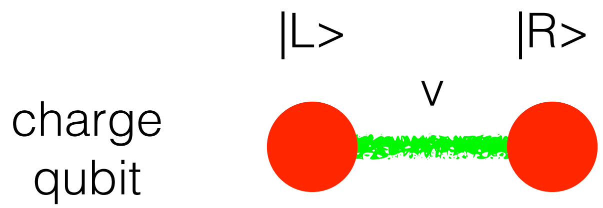
$$|\psi_0\rangle = \alpha\,|L\rangle + \beta\,|R\rangle$$

$$H = \begin{pmatrix} \epsilon & v \\ v & -\epsilon \end{pmatrix}$$

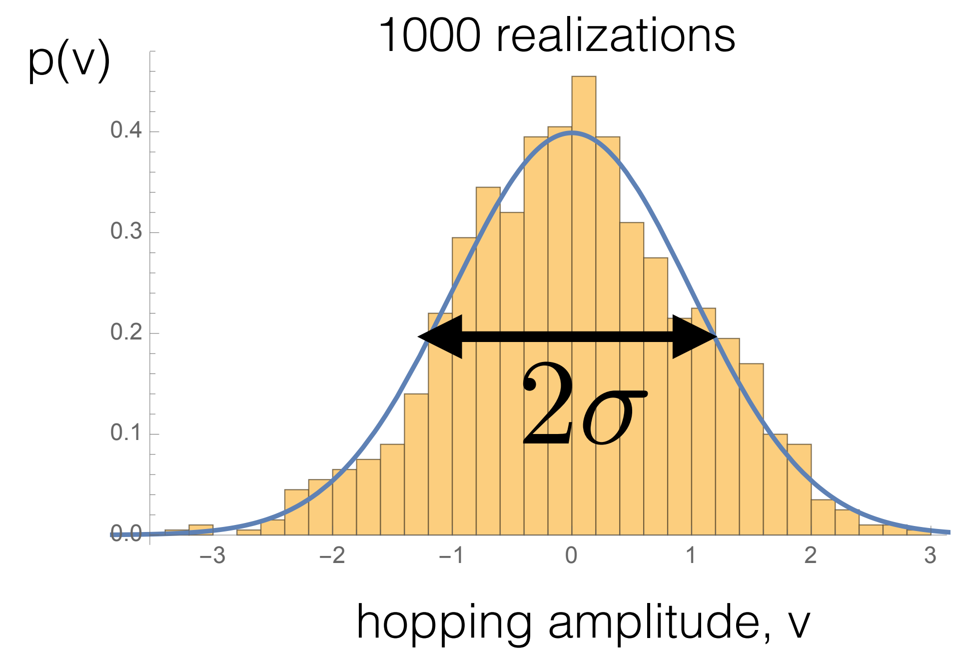
v: hopping amplitude
ϵ: on-site energy difference

To preserve the state, set $\epsilon = v = 0$.

Hopping noise erases information in the charge qubit



v: quasistatic Gaussian random hopping amplitude

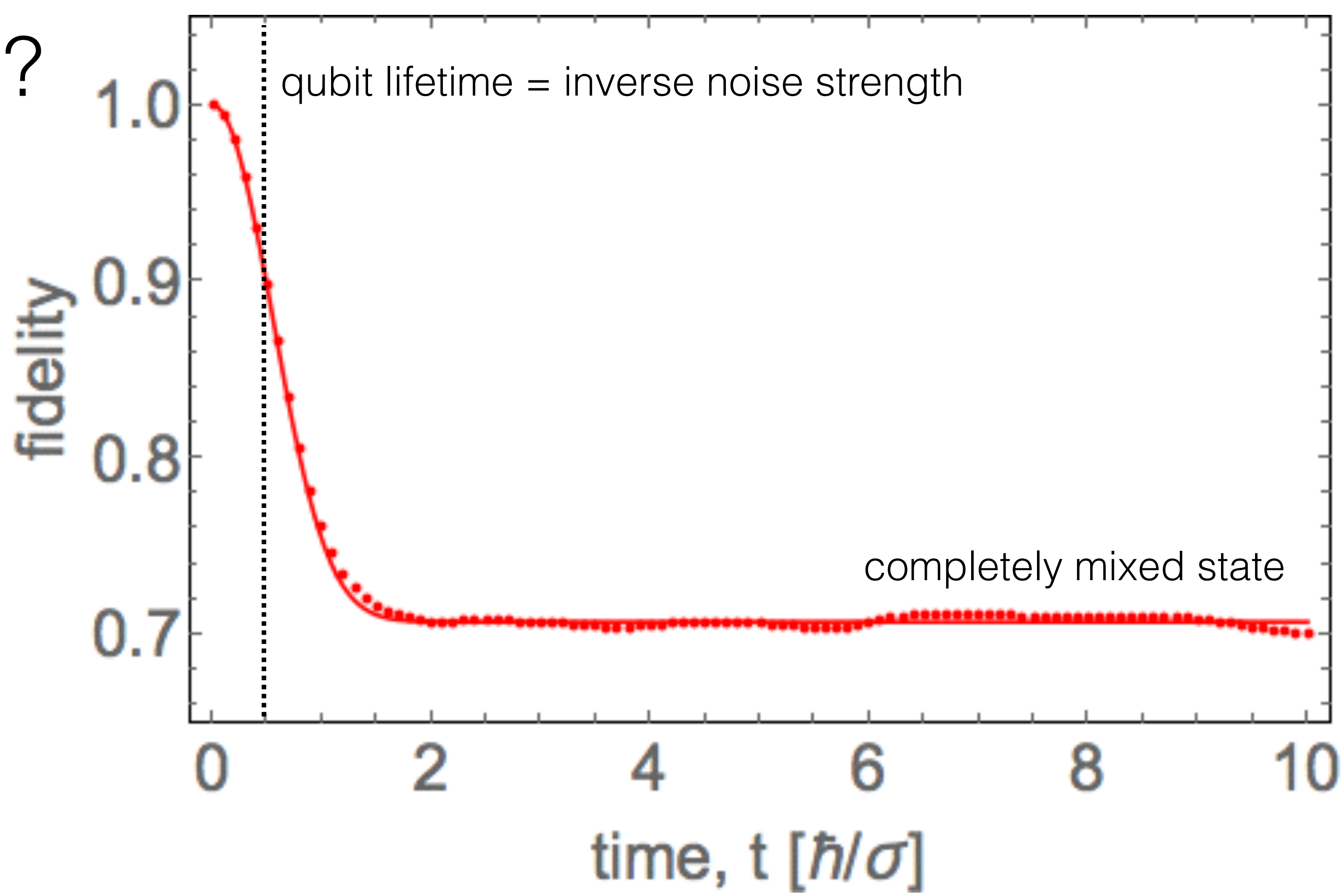


use initial state |L>

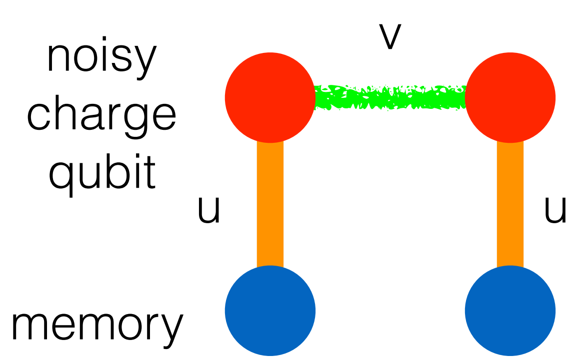
how well is it preserved?

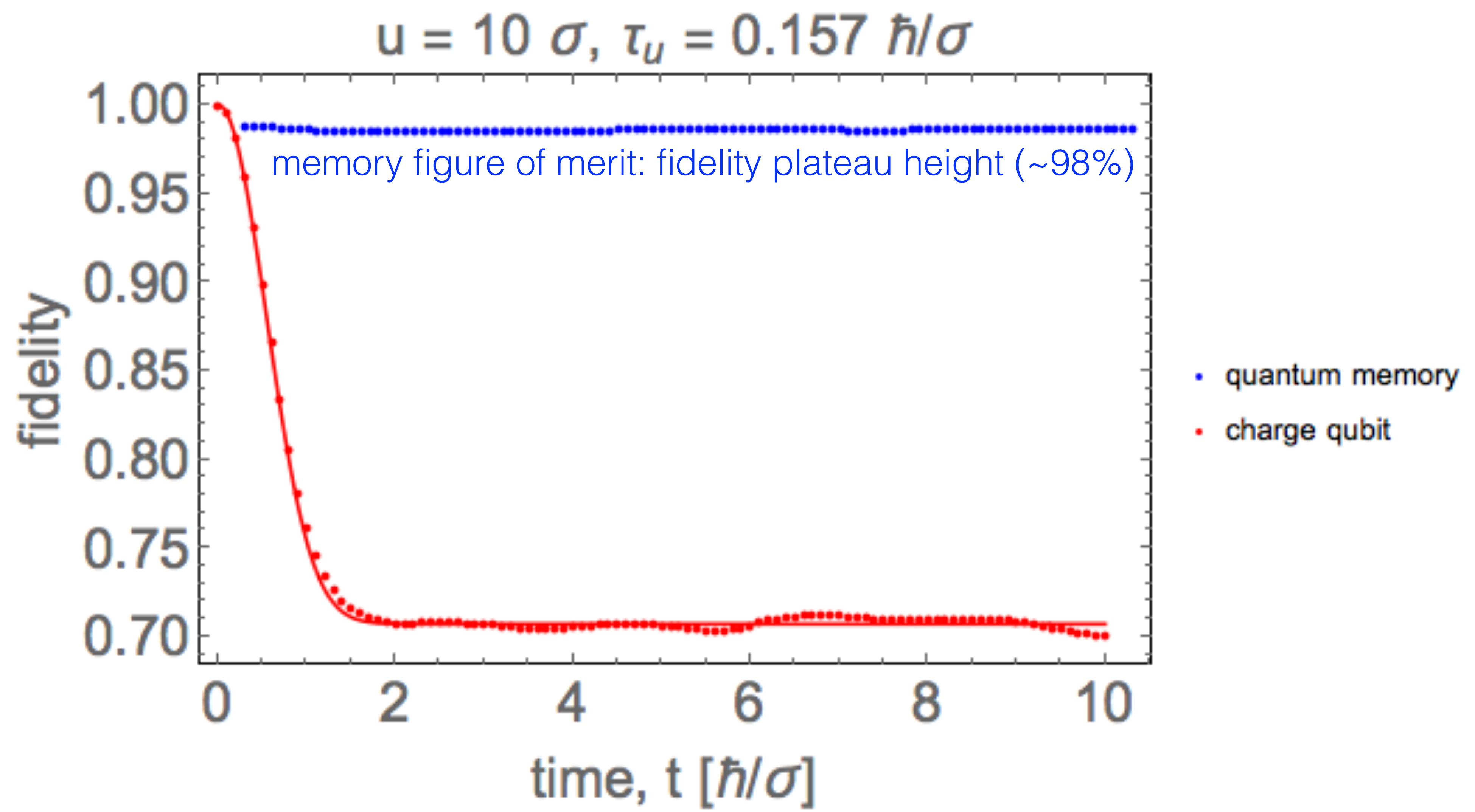
fidelity:

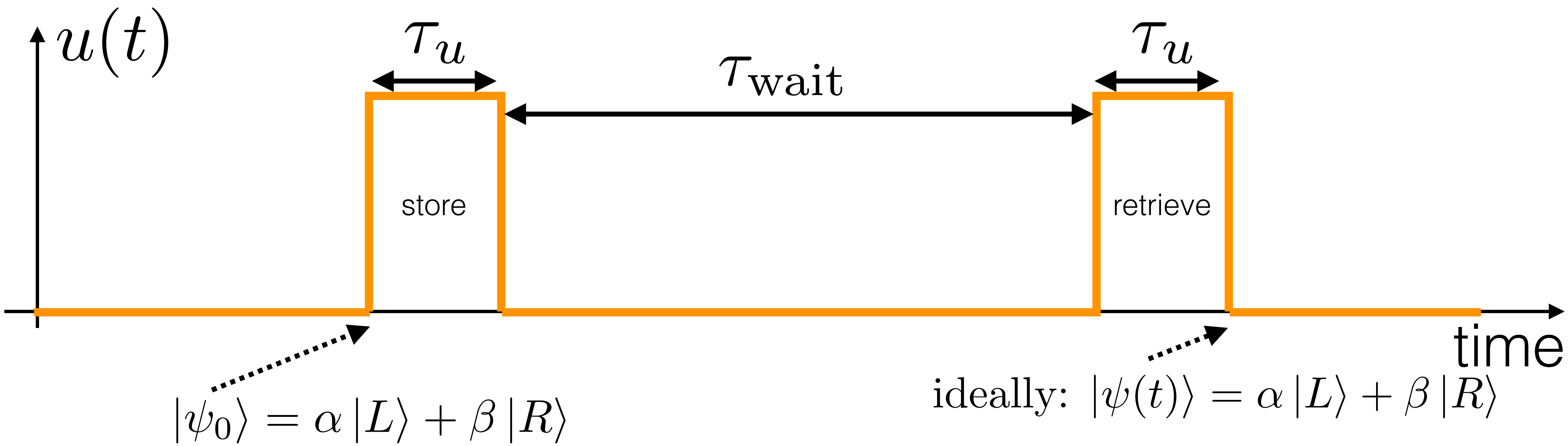
$$F(t) = \sqrt{\int_{-\infty}^{\infty} dv\, p(v) \left| \langle L | \psi^{(v)}(t) \rangle \right|^2}$$



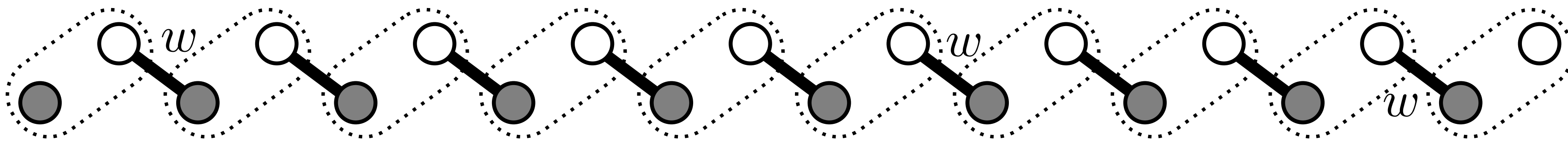
Information can survive if transferred to a less noisy qubit

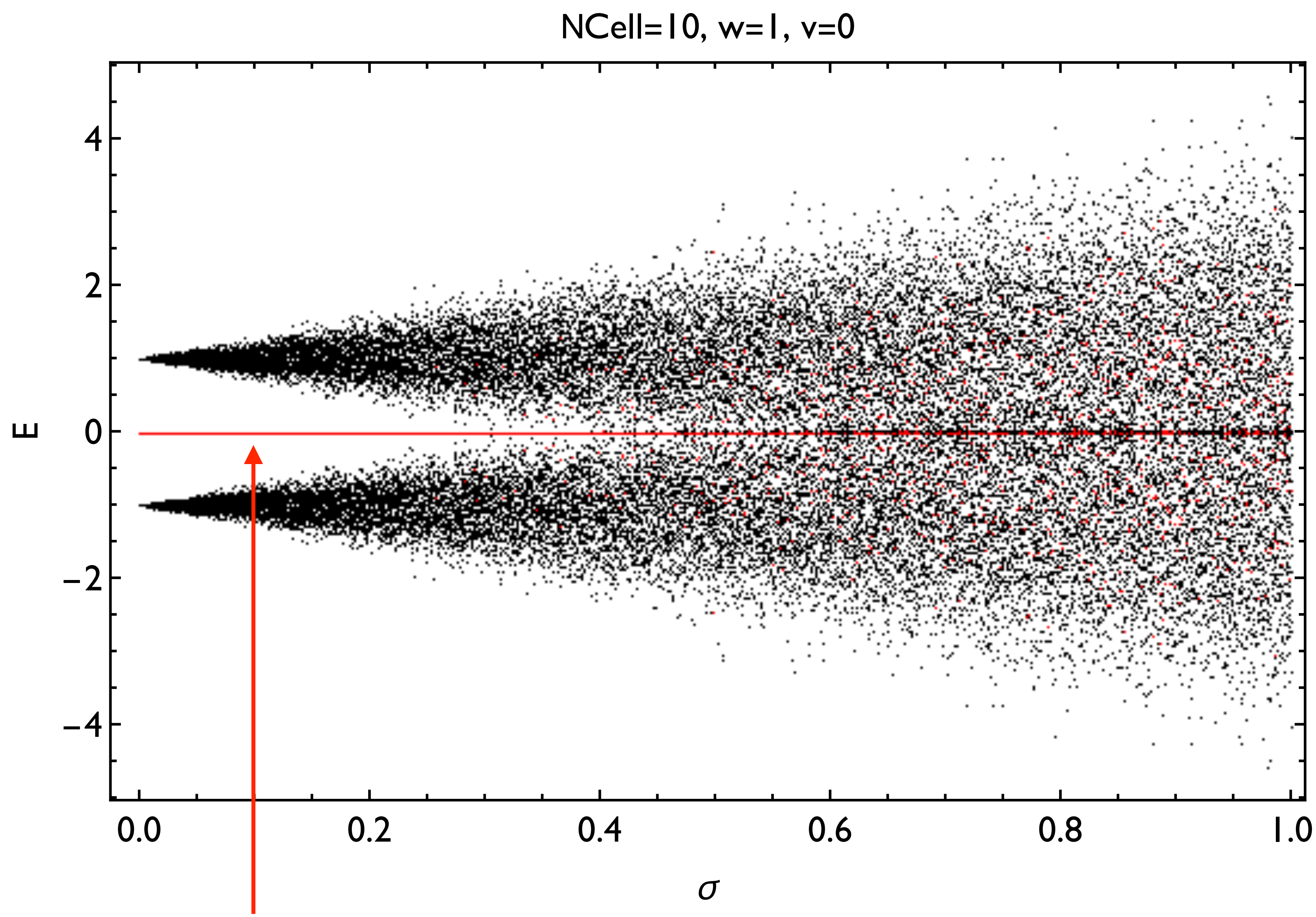






Zero-energy SSH states are protected from hopping noise

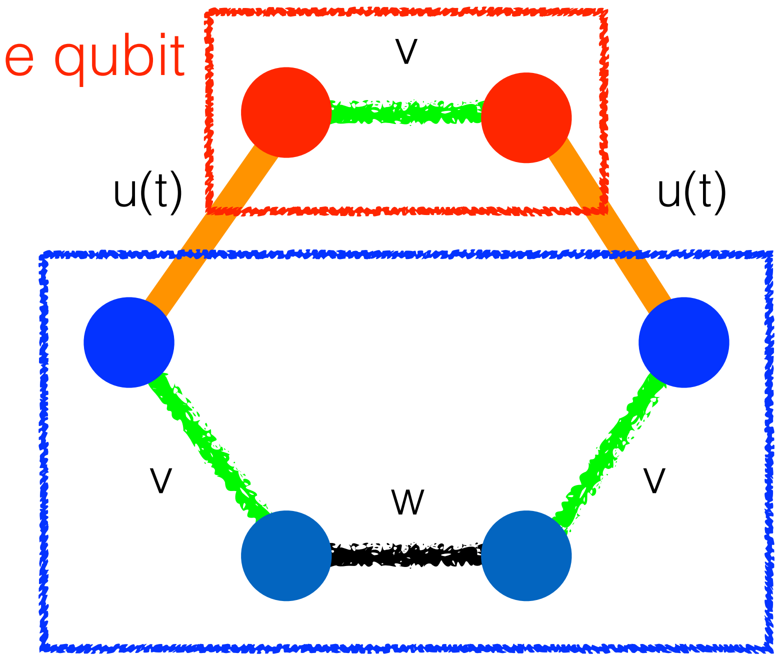




zero-energy edge states survive hopping disorder

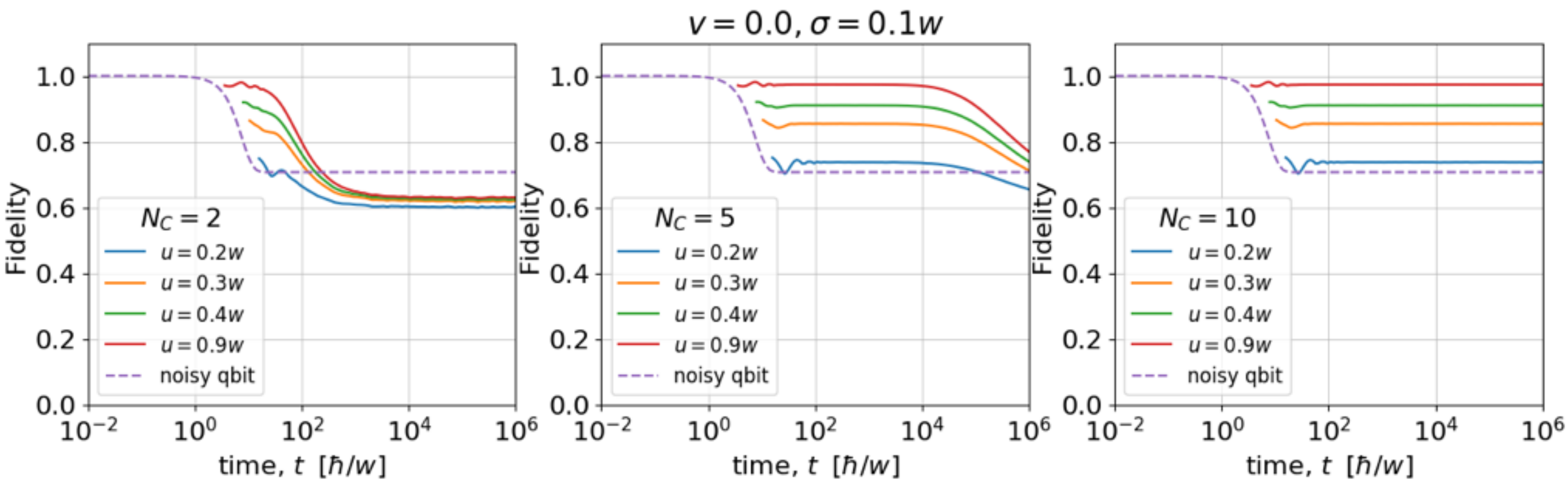
SSH chain with hopping noise is a good quantum memory

noisy charge qubit



noisy SSH chain
serving as a noiseless memory

all hoppings v, w
subject to the same
amount of noise

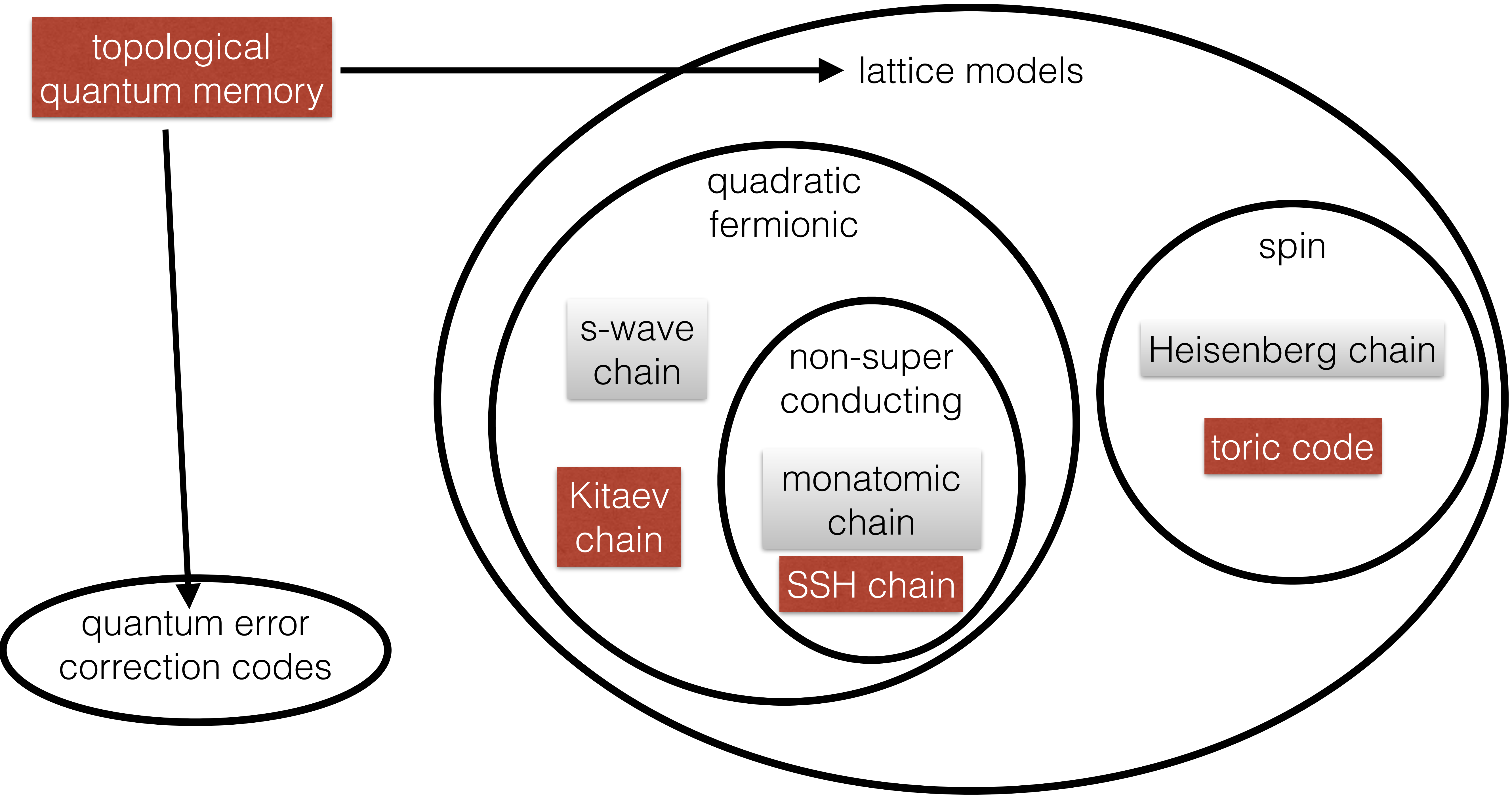


memory figures of merit: height & duration of fidelity plateau

Part 2
A topological quantum memory

An almost noiseless qubit is obtained by putting together many noisy qubits

Why do you call SSH a `topological' quantum memory?



1. GS can be degenerate if real-space lattice is compact
2. topology of real-space lattice => degree of GS degeneracy
3. size-protected GS degeneracy
4. (Kitaev chain, SSH: symmetry-protected GS degeneracy)

Why do you call SSH a `topological' quantum memory?

Short version of answer #1: because the Kitaev chain is called a topological quantum bit/ memory, and the SSH chain has the same properties (with particle-hole ->chiral)

Answer #2: because it is a quantum memory based on a topological insulator

Answer #3: hope more people read the abstract if `topological' is in the title

`topological quantum memory'

`size- and symmetry-protected quantum memory'

Can one realize such a topological memory?

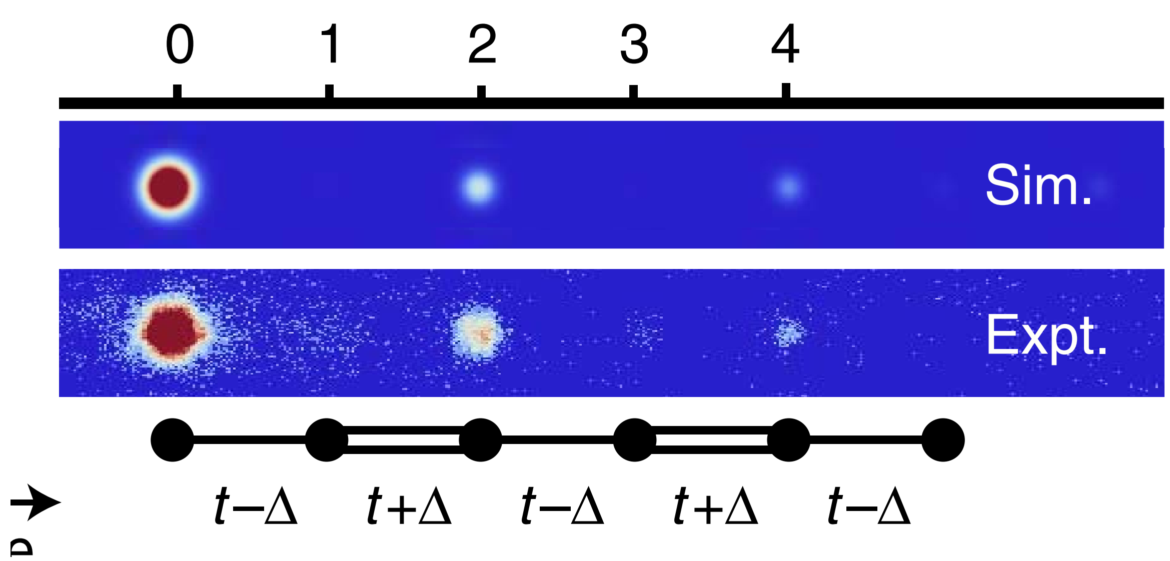
SSH model with cold atoms

ARTICLE

Received 27 Jul 2016 | Accepted 17 Nov 2016 | Published 23 Dec 2016 DOI: 10.1038/ncomms13986 OPEN

Observation of the topological soliton state in the Su-Schrieffer-Heeger model

Eric J. Meier[1], Fangzhao Alex An[1] & Bryce Gadway[1]

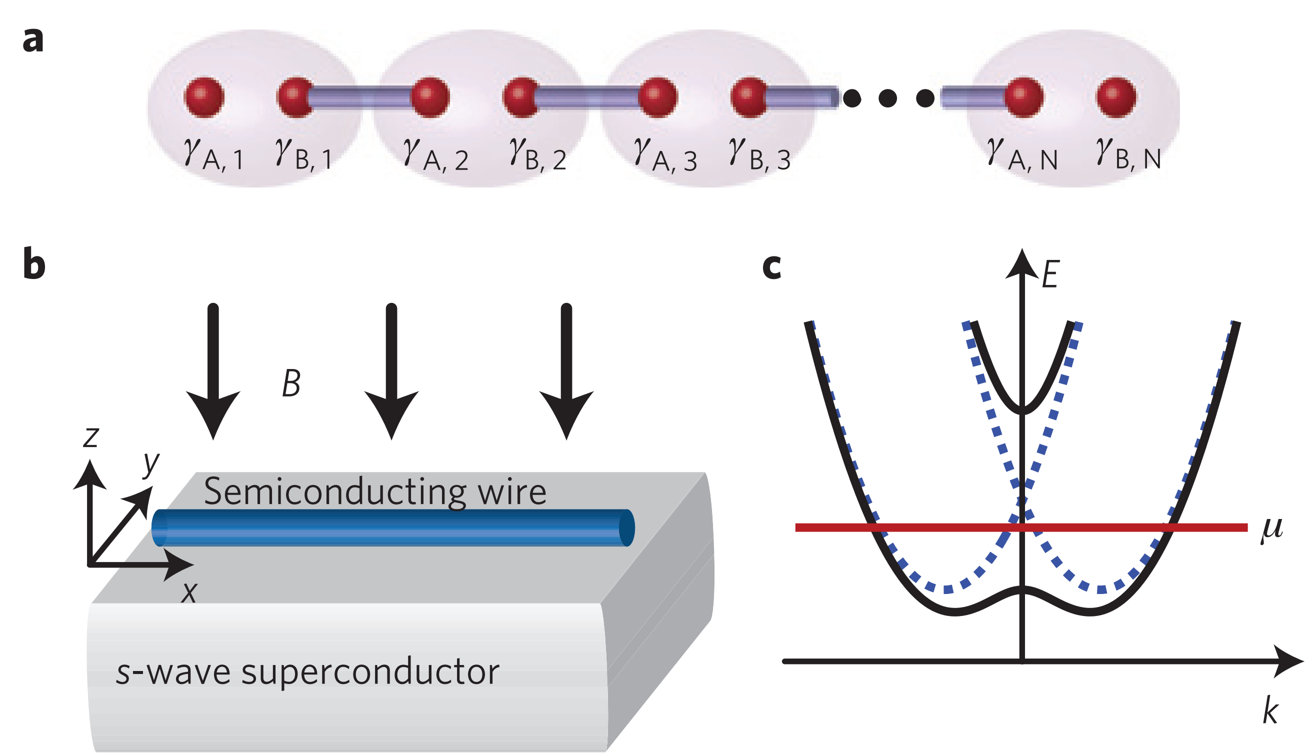


topological superconductors (see Prof. Ando's talk)

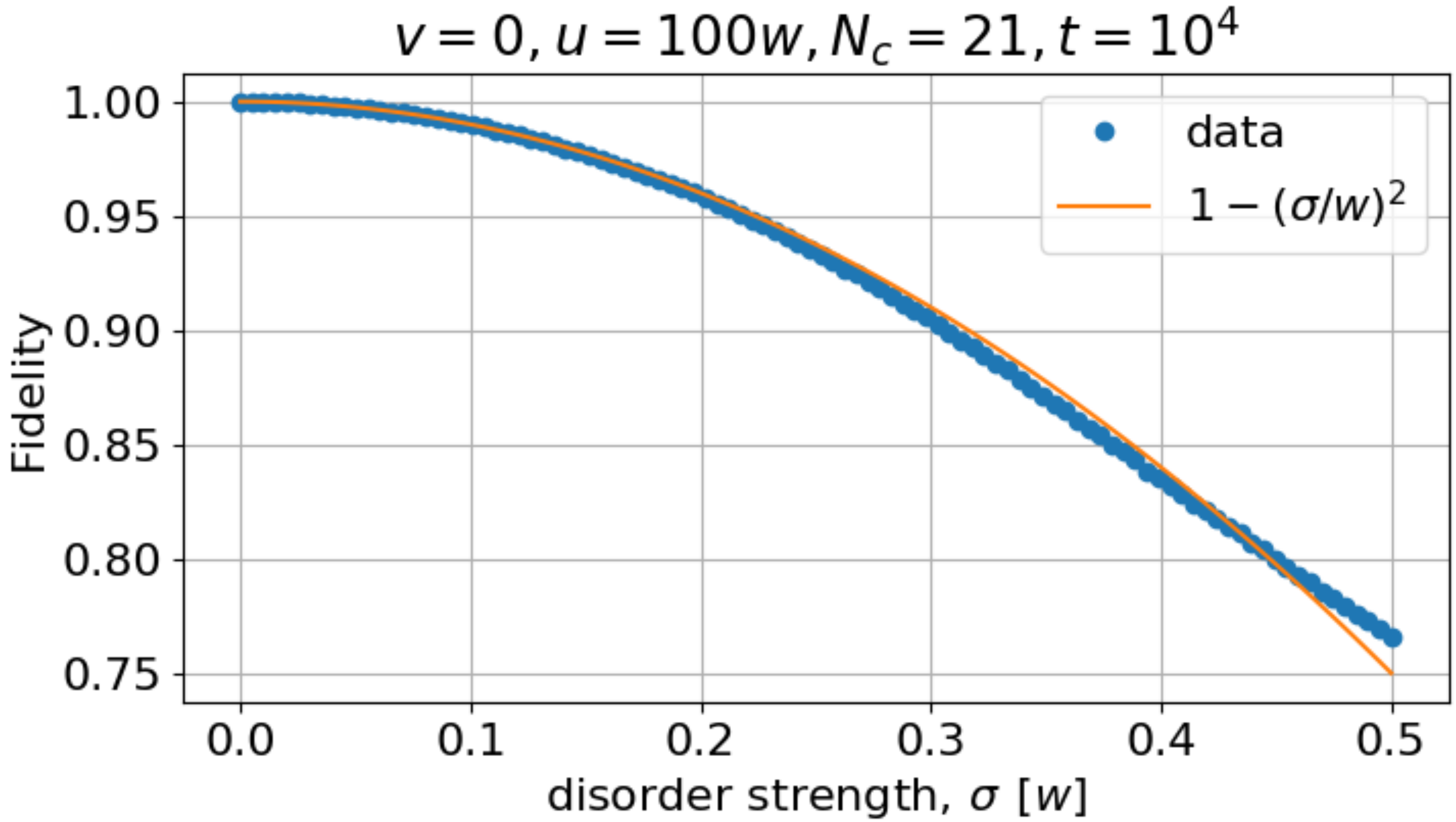
ARTICLES
PUBLISHED ONLINE: 13 FEBRUARY 2011 | DOI: 10.1038/NPHYS1915
nature physics

Non-Abelian statistics and topological quantum information processing in 1D wire networks

Jason Alicea[1★], Yuval Oreg[2], Gil Refael[3], Felix von Oppen[4] and Matthew P. A. Fisher[3,5]



Is the noisy SSH memory perfect?



Yes and no:
fidelity plateau duration can be increased arbitrarily
fidelity plateau height cannot
Reason: noise-induced uncontrolled hybridization bw 1A and 2A

The expectation value of the minigap can be expressed as

$$\mathbb{E}(\Delta) = \frac{2}{w^{N-1}} \left[v \,\mathrm{Erf}\left(\frac{v}{\sqrt{2}\sigma} \right) + \sqrt{\frac{2}{\pi}} \sigma e^{-\frac{v^2}{2\sigma^2}} \right]^N$$

In the clear case this can be simplified as

$$\mathbb{E}(\Delta) = \frac{2\, v^N}{w^{N-1}},$$

whereas in the full dimerized limit, i.e. $v = 0$, we can obtain

$$\mathbb{E}(\Delta) = \frac{2}{w^{N-1}} \left(\sqrt{\frac{2}{\pi}} \sigma \right)^N .$$

```
In[2010]:= 2 * (Sqrt[2 / π] 0.1) ^ 2
            2 * (Sqrt[2 / π] 0.1) ^ 5
            2 * (Sqrt[2 / π] 0.1) ^ 10

Out[2010]= 0.0127324

Out[2011]= 6.46741 × 10^-6

Out[2012]= 2.09137 × 10^-11
```